Hint Sheet: 7.5

1. Let $u = \sin(x)$
2. $\dfrac{1-\cos(x)}{\sin(x)} = \dfrac{1-\cos(x)}{\sin(x)}\dfrac{1+\cos(x)}{1+\cos(x)} = \dfrac{\sin(x)}{1+\cos(x)}$
3. Let $u = \tan^{-1} y$ and do u, du substitution.
4. Integrate by parts with x^3 in the middle column.
5. Pull out $\cos(x)$ to keep with dx.
6. Let $u = \cos(x)$ and do u, du substitution.
7. Let $x = 3\sin(\theta)$
8. This is very similar to a $\sin(\theta)$ substitution. If we let $u = x^2$, we'll get the right form.
9. Because of the x in the numerator, try $u = 1 - x^2$.
10. Let $x = \sin(\theta)$.
11. Partial Fractions: $\frac{A}{t-3} + \frac{B}{(t-3)^2} = \frac{2t}{(t-3)^2}$
12. Partial Fractions: $\frac{A}{x-5} + \frac{B}{x+1} = \frac{x-1}{(x-5)(x+1)}$
13. Complete the square: $x^2 - 4x + 5 = (x-2)^2 + 1$, then let $u = x - 2$.
14. First, let $u = x^2$, then complete the square so that the denominator is: $(u + \frac{1}{2})^2 + \frac{3}{4}$. Finally, let $w = u + \frac{1}{2}$ and see the $w^2 + a^2$ in the denominator.
15. First write e^{x+e^x} as $e^x e^{e^x}$. Then let $u = e^x$.
16. Let $u = \sqrt[3]{x}$. Then $u^3 = x$, and $3u\ du = dx$, and substitute. Integrate by parts using a table.
17. Use integration by parts with $u = \ln(1+x^2)$ and $dv = dx$.
18. You probably tried $u = \ln(x)$ and got to the integral of $\frac{\sqrt{1+u}}{u}$. Now let $w = \sqrt{1+u}$ so that $w^2 = 1 + u$, $2w\ dw = du$, etc.
19. Integrate by parts using a table with t^3 in the middle column.
20. Integrate by parts once using $u = \sin^{-1}(x)$, $dv = x$. To integrate what's remaining after the first step, use $x = \sin(\theta)$.
21. Let $u = 1 + \sqrt{x}$ so that $x = (u-1)^2$, and $dx = 2(u-1)\ du$
22. Expand by multiplying $\sqrt{z}$ through.
23. First, long division yields: $3 + \frac{6x+22}{x^2-2x-8}$ To integrate the second term, using partial fractions.
24. Let $u = x^3 - 2x - 8$.
25. Let $u = \ln(\sin(x))$. Note that $du = \cot(x)\ dx$.
26. Let $u = \sqrt{at}$ so that $u^2 = at$ and $2u\ du = a \cdot dt$, then integrate by parts.
27. By factoring, $x^3 + x^2 - 2x = x(x+2)(x-2)$, which is negative from -3 to -2, and from 0 to 1. So separate the integral into 4 pieces.
28. First complete the square under the radical sign: $1 + x - x^2 = \frac{5}{4} - (x - \frac{1}{2})^2$. Now let $u = x - \frac{1}{2}$ and use a $\sin(\theta)$ substitution.
29. First multiply the top and bottom by $\sqrt{1+x}$ as in Example 5.
30. Let $u = \sqrt{2x-1}$, so $u^2 = 2x - 1$, and $2u\ du = 2\ dx$, $2x + 3 = u^2 + 4$, etc.
31. Do long division first, so $\frac{3w-1}{w+2} = 3 - \frac{7}{w+2}$
32. $x^3 - 8 = (x-2)(x^2+2x+4)$, then do partial fractions.
33. Integrate by parts twice to get $\int e^{2x}\sin(3x)\ dx$ on both sides of the equation, etc.
34. Let $u = \cos^2(x)$. Recall that $2\sin(x)\cos(x) = \sin(2x)$.
35. This is something we didn't talk about in class (Odd functions), so skip it (you can look it up, 5.5.7(b)))
36. Use 7.2.2(a) (Table 2)
37. Write in terms of sines and cosines and simplify.
38. Pull out a $\sec^2(x)$ to keep with dx.
39. Let $u = 1 - x^2$. Once you've written the integral in u, you might find $w = sqrtu$, $w^2 = u$, $2w\ dw = du$ a handy next substitution.
40. Complete the square: $4y^2 - 4y - 3 = (2y-1)^2 - 2^2$
41. Integrate by parts, noting that the antiderivative of $\tan^2(\theta)$ is the antiderivative of $\sec^2(\theta) - 1$ which is $\tan(\theta) - \theta$.
42. As in (41), take the antiderivative of $\tan^2(4x)$ by first writing $\sec^2(4x) - 1$.
43. Let $t = x^3$, so that $dt = 3x^2\ dx$. Note that that leaves another x^3 unaccounted for. Substitute t for that x^3, also.
44. Let $u = e^x$ so that $\ln(u) = x$, and $\frac{1}{u}\ du = dx$. Now do partial fractions on what remains.
45. Split into two fractions: $\frac{x}{x^2+a^2} + \frac{a}{x^2+a^2}$.
46. Let $u = x^2$, then partial fractions.
47. Use half-angle identities, and multiply everything out (kind of long!)
48. Integrate by parts with $u = \tan^{-1}(x)$, $dv = x^2\ dx$.
49. Let $u = \sqrt{4x+1}$, then use partial fractions.
50. Let $u = \sqrt{4x+1}$, then use partial fractions (you'll have 4 constants to solve for).
51. Let $2x = \tan(\theta)$.
52. Trick: $\frac{1}{x(x^4+1)} = \frac{x}{x^2(x^4+1)}$, Let $u = x^2$. Note: You don't need to do the trick, it just makes it a little easier.
53. Skip this one (unless you want to read about hyperbolic functions- look it up in the Appendix).
54. Multiply it out.
55. Let $u = \sqrt{x+1}$
56. Let $t = \sqrt{x^2-1}$, arriving at $\int \ln(t^2+1)\ dt$ Use integration by parts.
57. Let $u = \sqrt[3]{x+c}$
58. Integrate by parts first, then do long division.
59. Let $u = e^x$. Note that $x = \ln(u)$.
60. Let $u = \sqrt[3]{x}$.
61. Let $u = x^5$.
62. Let $u = x + 1$.
63. Rewrite in sines and cosines, pull out a cosine.
64. Let $u = \tan(x)$.
65. Multiply numerator and denominator by $\sqrt{x+1} - \sqrt{x}$.
66. Long division followed by partial fractions.
67. Let $u = \sqrt{t}$, followed by integration by parts.
68. Let $u = e^x$. Again, note that $x = \ln(u)$.
69. Let $u = e^x$.
70. Try integration by parts with $u = \ln(x+1)$.
71. Partial Fractions (4 constants).
72. $u = \sqrt[6]{t}$. Note $u^6 = t$.
73. Partial fractions (3 constants).
74. Let $u = e^x$.
75. First use Table 2 to re-write $\sin(2x)\cos(3x)$. Then multiply out and use Table 2 again.
76. Integration by parts.