Functions of Several Variables

Understanding from three points of view: Geometric, Algebraic, and a little Numerical

$$z = f(x, y)$$

$$w = f(x, y, z)$$

Natural Domain

The domain of $z = f(x, y)$ is the set of (x, y) for which f is defined.

Natural Domain

The domain of $z = f(x, y)$ is the set of (x, y) for which f is defined.

Example

Find the domain: $f(x, y) = \sqrt{y} + \sqrt{25 - x^2 - y^2}$

Natural Domain

The domain of $z = f(x, y)$ is the set of (x, y) for which f is defined.

Example

Find the domain: $f(x, y) = \sqrt{y} + \sqrt{25 - x^2 - y^2}$

- From $\sqrt{y}$, $y \geq 0$.

Natural Domain

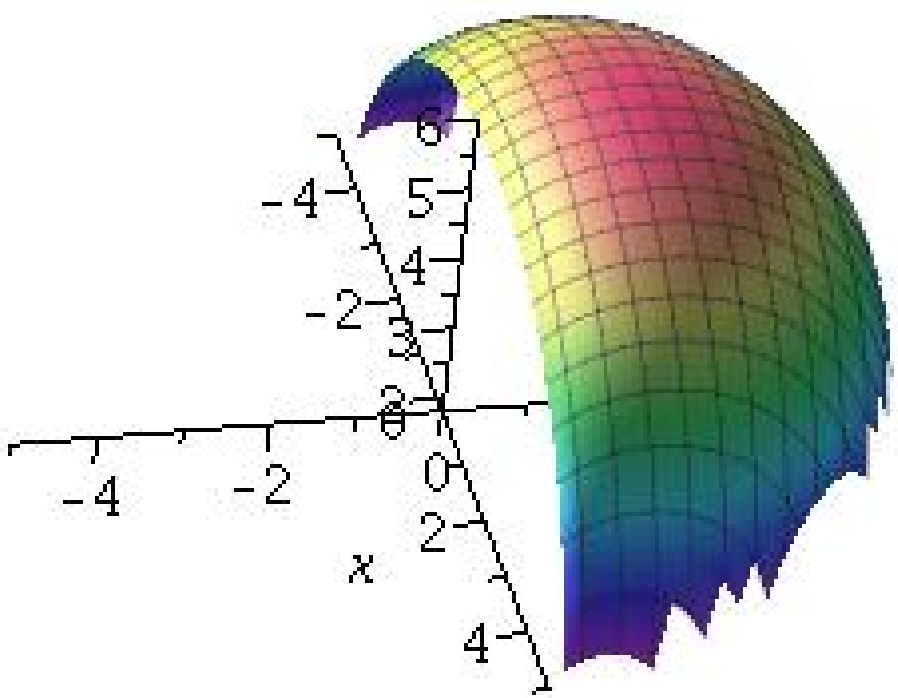
The domain of $z = f(x, y)$ is the set of (x, y) for which f is defined.

Example

Find the domain: $f(x, y) = \sqrt{y} + \sqrt{25 - x^2 - y^2}$

- From $\sqrt{y}$, $y \geq 0$.
- From the square root, $x^2 + y^2 \leq 25$.

Plot the domain, then we'll plot the surface.


-4
-2
x
-4
-2
6
5
4
3
2
1
0
2
4

Example: Numerical Chart

Apparent temperature, given actual temp and humidity is below.

Relative humidity (%)

Actual temperature (°F) T \ h	20	30	40	50	60	70
80	77	78	79	81	82	83
85	82	84	86	88	90	93
90	87	90	93	96	100	106
95	93	96	101	107	114	124
100	99	104	110	120	132	144

Questions for the numerical chart:

1. What is the value of $f(95, 70)$? What does it mean?

Questions for the numerical chart:

1. What is the value of $f(95, 70)$? What does it mean?
2. For what h is $f(90, h) = 100$?

Questions for the numerical chart:

1. What is the value of $f(95, 70)$? What does it mean?
2. For what h is $f(90, h) = 100$?
3. For what T is $f(T, 50) = 88$?

Questions for the numerical chart:

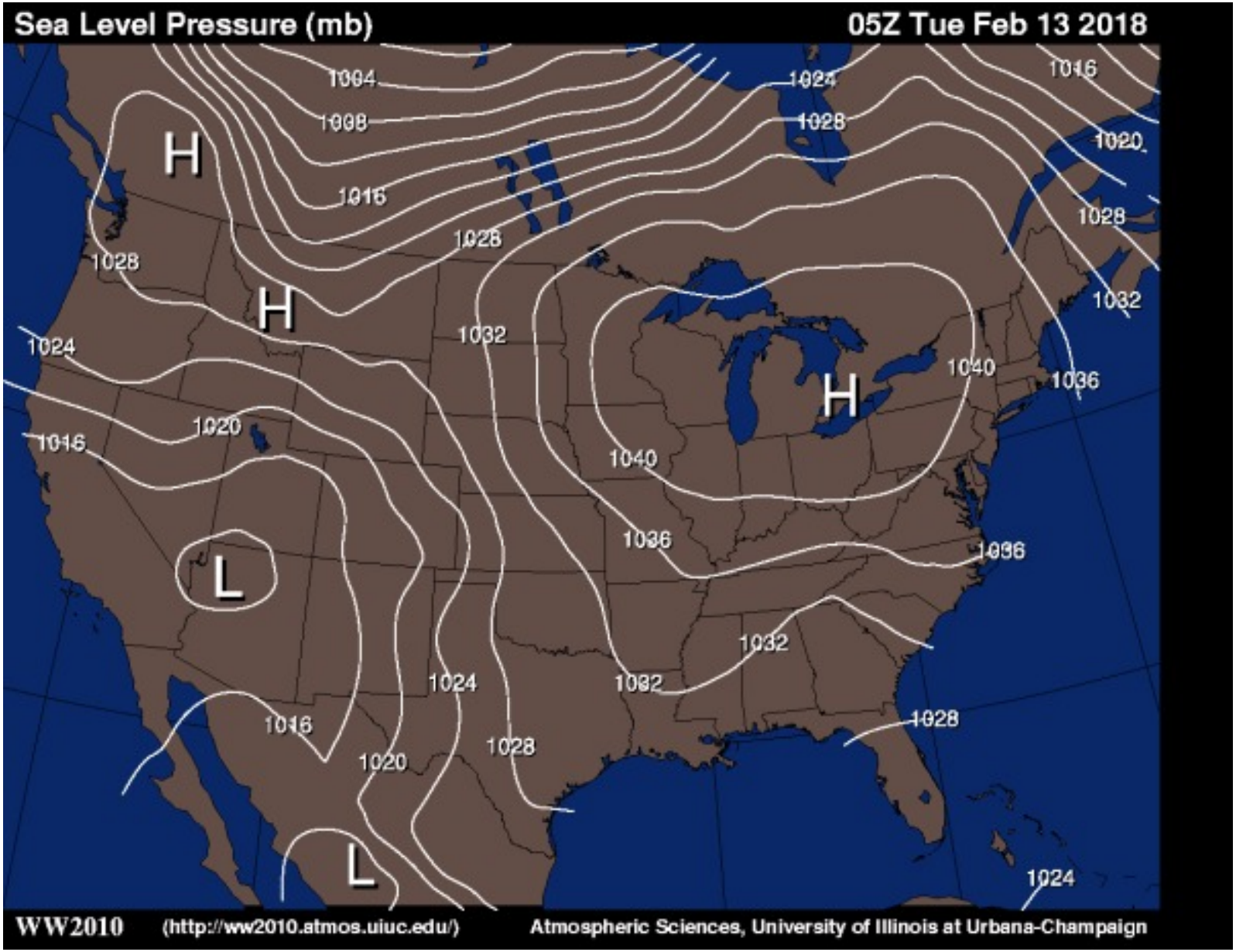
1. What is the value of $f(95, 70)$? What does it mean?
2. For what h is $f(90, h) = 100$?
3. For what T is $f(T, 50) = 88$?
4. What is the meaning of the function $I(h) = f(80, h)$?

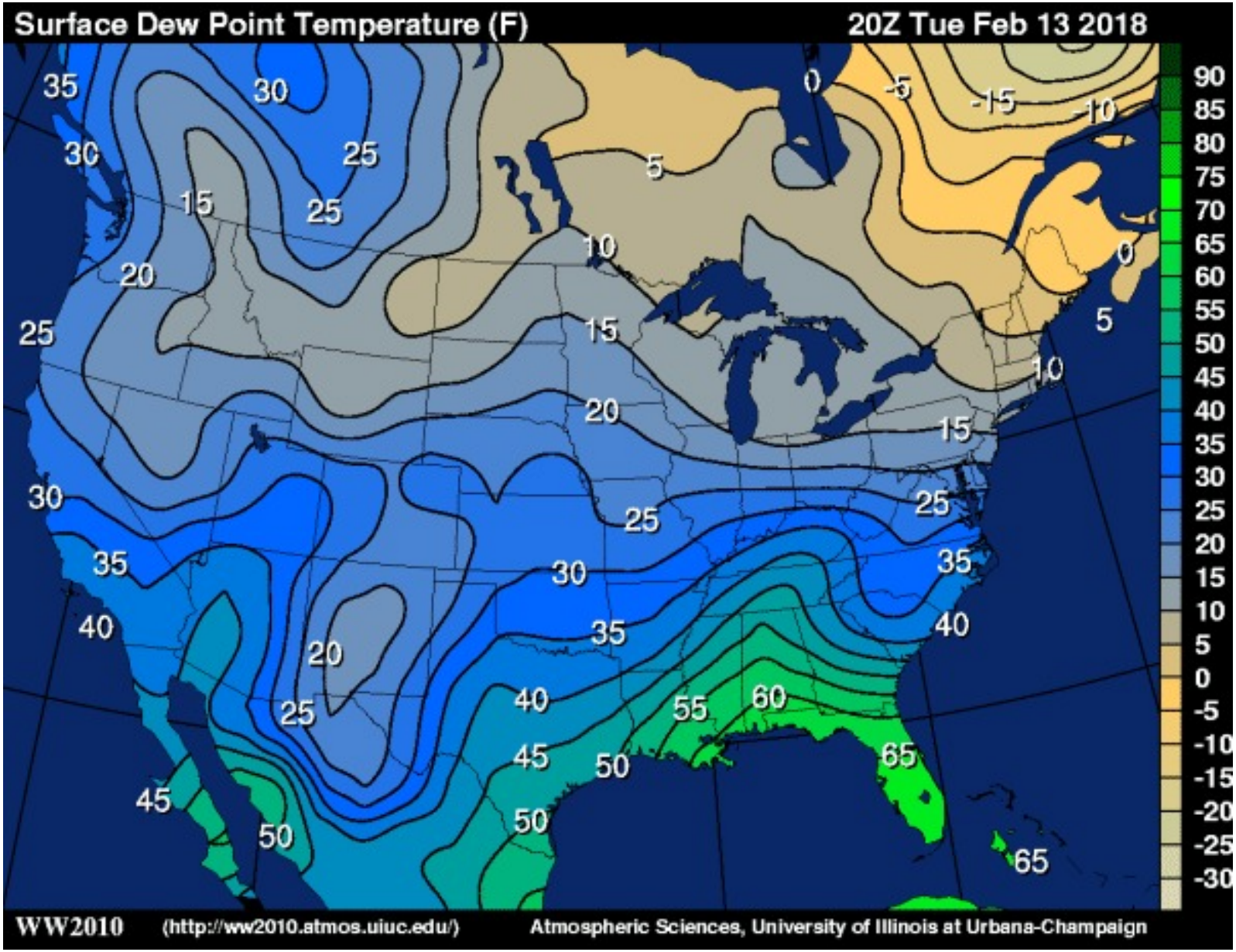
Level Curves

Given $z = f(x, y)$, we can fix the value of z, and graph **level curves** of the form:

$$f(x, y) = k$$

In weather maps, the level curves for pressure are called **isobars**


Sea Level Pressure (mb)
05Z Tue Feb 13 2018
1004
1008
1012
1016
1020
1024
1028
1032
1036
1040
H
H
H
L
L
1028
1024
1016
1020
1016
1024
1028
1032
1040
1036
1036
1032
1028
1024
1028
1032
1036
1040
1036
1032
1028
1024
1020
1016
1016
1024
1028
1032
1036
WW2010
(http://ww2010.atmos.uiuc.edu/)
Atmospheric Sciences, University of Illinois at Urbana-Champaign


Surface Dew Point Temperature (F)
20Z Tue Feb 13 2018
WW2010
(http://ww2010.atmos.uiuc.edu/)
Atmospheric Sciences, University of Illinois at Urbana-Champaign

Level Curves

The level curves should be equal distance apart. For example, in the pressure map, the pressure levels were

$$1004, 1008, 1012, 1016, \cdots$$

If a surface is steep, the level curves are tightly packed.

Level Curves

The level curves should be equal distance apart. For example, in the pressure map, the pressure levels were

$$1004, 1008, 1012, 1016, \cdots$$

If a surface is steep, the level curves are tightly packed.
If a surface is shallow, the level curves are far apart.

Level Curves

The level curves should be equal distance apart. For example, in the pressure map, the pressure levels were

$$1004, 1008, 1012, 1016, \cdots$$

If a surface is steep, the level curves are tightly packed.
If a surface is shallow, the level curves are far apart.

The level curves can be shown in two or three dimensions
(See Sect 14.1, figure animations from the text)