Topics for Exam 1, Calculus 3

Parametric Curves

- Define parametric curves in the plane. Special example: Circles in the plane.
- Define first and second derivatives.
- Be able to go back and forth between the pair of graphs $((t, x), (t, y))$ and the graph in the $xy-$plane.
- Algebraically switch between representations (from parametric to (x, y) and vice-versa). In particular, $y = f(x)$.
- Compute the tangent line to a given parametric curve.
- Compute the area of a curve given in parametric form.
- Compute the arc length of a curve given in parametric form (esp the setup).
- Nothing on the surface area will be on the exam (from 10.3).

Polar Coordinates

- Ideas: From a right triangle, derive the conversion formulas needed to go back and forth between (x, y) and (r, θ).
- By interpreting polar curve $r = F(\theta) \Leftrightarrow x = f(\theta)$, $y = g(\theta)$ as parameteric be able to apply the notes on parametric curves (esp tangent line).
- This is a good time to review your basic trigonometry (definitions, unit circle, basic graphs, 30-60-90 triangle, 45-45-90 triangle).

Introduction to 3d and Vectors.

- Ideas: Introduce the three-dimensional coordinate system, right hand rule for setting up the coordinate system, coordinate planes, distance formula. Be familiar with the formula for a sphere (be able to complete the square!) and recognize/sketch various planes in 3d.
- Know that there are three ways we look at/define vectors (free, position and rooted).
- Important definitions: components (of a vector), position vector (or position representation of a vector), magnitude (or length) of a vector, the standard basis vectors $(\vec{i}, \vec{j}, \vec{k})$, *unit* vector
- Know the operations: scalar multiplication and vector addition/subtraction mean both graphically and algebraically. Relatedly, know how to construct vectors (head$-$tail, for example)
- New operation: dot product. Dot product is used to get angles and is used to compute compute projections. Relatedly, know: the magnitude in terms of the dot product, use the dot product to find the angle between vectors and determine if vectors are orthogonal (a.k.a. perpendicular). Know the direction cosines and direction angles. Be able to use the properties of the dot product listed on pg. 801.
- Work: $W = \vec{F} \cdot \vec{D}$. For now, I won't ask you about torque.
- New operation: cross product. Know the definition, and that the cross product is computed so that the right-hand rule applies. Understand that the cross product is only defined for vectors in $\mathbb{R}^3$.

- Formulas: Determinants ($2 \times 2, 3 \times 3$), computing the scalar tripe product, the relationship with the $\sin(\theta)$.
- Skills: How to tell if vectors are parallel or coplanar. Construct a vector perpendicular to two others.
- Algebraically manipulate expressions involving the cross product. In Theorem 11, pg. 812, be able to use formulae (1)-(4). The others will also be used in the future, but they are not intuitively clear, and I'll give you reminders of those.
- Area and Volume: Find the area of a parallelogram in two dimensions (defined by two vectors), and the area in three dimensions (again defined by two vectors). Find the volume of a parallelepiped in three dimensions (defined by three vectors).

Lines, Planes, and some Surfaces

- A line is defined by a point (on the line) and a direction. Algebraically, we have three ways of expressing a line: Parametric equations, vector equation, and symmetric equations. Be able to convert from one to the other.
- A plane is defined by a point (in the plane), and two directions, although it is better algebraically to think of a plane as being defined by a point an a normal vector to the plane. Algebraically express the equation of a plane as either a scalar equation ($A(x-x_0)+B(y-y_0)+C(z-z_0)=0$) or parametrically as: $\mathbf{x}_0 + s\mathbf{u} + t\mathbf{v}$.
- Definitions: Skew lines, parallel lines, parallel planes, the angle between planes.
- Be able to compute the distance between a point and a plane, between two planes, between two skew lines. In particular, be able to derive the formula for the distance between a point and a plane using the projection (see Fig 12, p. 822).
- Be able to get an idea of the shape of basic objects in three dimensions by plotting traces in two dimensional cross sections. In particular, know these:

  $$x^2 + y^2 + z^2 = 1 \qquad z = x^2 + y^2 \qquad z = x^2 - y^2$$

  The first two come up a lot. The third may be unusual to us, but also comes up a lot later (the origin is neither a high point nor a low point of the graph)

  Do be able to match formulas to graphs.