

Example: Exercise 17, Section 16.9

> #Define the vector field:

> with(VectorCalculus): with(Student[VectorCalculus]): with
(plots):

> F:=VectorField(<x*z^2, (1/2)*y^3+tan(z), x^2*z+y^2>);

$$F := (xz^2)\bar{e}_x + \left(\frac{1}{2}y^3 + \tan(z)\right)\bar{e}_y + (x^2z + y^2)\bar{e}_z \quad (1)$$

> #Define the surface:

> S:=<sin(v)*sin(u), sin(v)*cos(u), cos(v)>; # u is between 0 and
2pi, v is 0 to Pi/2

$$S := (\sin(v)\sin(u))e_x + (\sin(v)\cos(u))e_y + (\cos(v))e_z \quad (2)$$

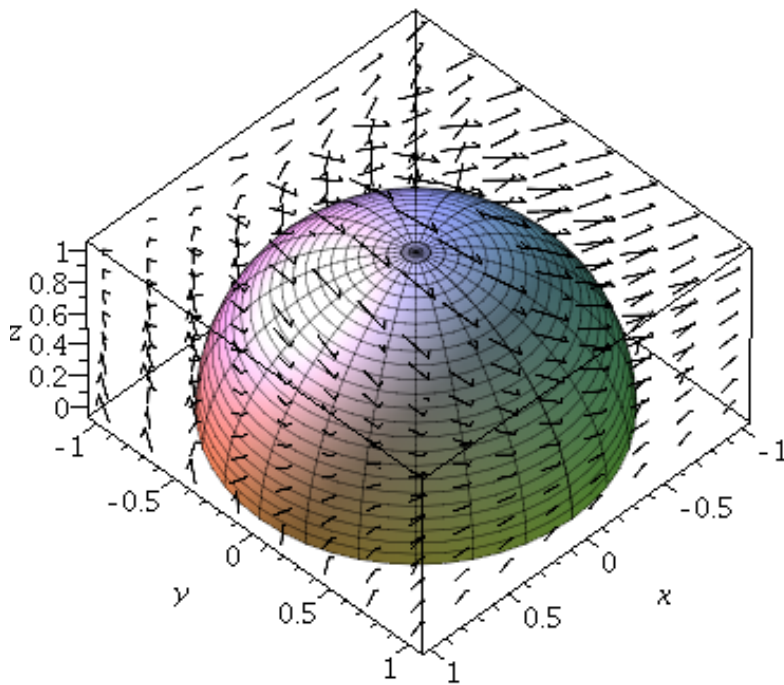
> A:=fieldplot3d(F,x=-1..1,y=-1..1,z=0..1,color=black,
fieldstrength=fixed);

$$A := \text{PLOT3D}(\dots) \quad (3)$$

> B:=plot3d(S,u=0..2*Pi,v=0..Pi/2);

$$B := \text{PLOT3D}(\dots) \quad (4)$$

> display3d(A,B,scaling=constrained);



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> #Compute the divergence (we're converting to spherical
    coordinates as well):
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> dF:=Divergence(F);
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$$dF := z^2 + \frac{3}{2} y^2 + x^2 \quad (5)$$

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> dG:=subs({x=r*sin(v)*cos(u),y=r*sin(v)*sin(u),z=r*cos(v)},dF);
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$$dG := r^2 \cos(v)^2 + \frac{3}{2} r^2 \sin(v)^2 \sin(u)^2 + r^2 \sin(v)^2 \cos(u)^2 \quad (6)$$

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> int(int(int(dG*r^2*sin(v),r=0..1),u=0..2*Pi),v=0..Pi/2);
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$$\frac{7}{15} \pi \quad (7)$$