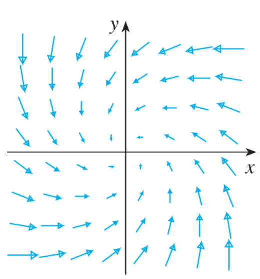
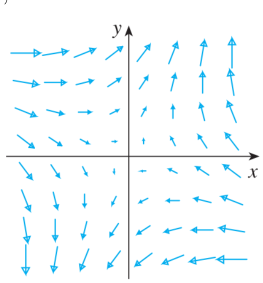
Final Exam Pack A

1. Short Answer/True or False. You may assume that all vectors are in $\mathbb{R}^3$.
   (a) $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.
   (b) $\frac{d}{dt}(\mathbf{u}(t) \times \mathbf{v}(t)) = \mathbf{u}'(t) \times \mathbf{v}'(t)$
   (c) There is a vector field whose curl is given by $\langle x, y, z \rangle$.
2. A constant force $\mathbf{F} = \langle 3, 5, 10 \rangle$ moves an object along the line segment from $(1, 0, 2)$ to $(5, 3, 8)$. Find the work done if the distance is in meters and the force is measured in newtons.
3. If $u = \sqrt{r^2 + s^2}$, $r = y + x\cos(t)$ and $s = x + y\cos(t)$, compute u_x and u_t at $x = 1, y = 2$ and $t = 0$.
4. Let vector $\mathbf{a} = \langle 1, 1, -2 \rangle$ and $\mathbf{b} = \langle 3, 2, -1 \rangle$.
   (a) Find the area of the parallelogram formed using $\mathbf{a}, \mathbf{b}$ (as position vectors).
   (b) Find $\text{Proj}_{\mathbf{b}}(\mathbf{a})$
5. Find the set of points for which $g(x, y) = \ln(x^2 + y^2 - 4)$ is continuous.
6. Find the equation of a plane (normal form, not parametric form), if the plane goes through points $(3, -1, 1)$, $(4, 0, 2)$ and $(6, 3, 1)$.
7. Suppose an object starts at the origin with initial velocity $\langle 1, -1, 3 \rangle$, and its acceleration is given by $\mathbf{a}(t) = \langle 6t, 12t^2, -6t \rangle$. Find its position function.
8. Let $f(x, y) = \sqrt[3]{x^3 + y^3}$. Is f differentiable at the origin? Hint: Compute $f_x(0, 0)$ using the *definition* and compare to the derivative using the regular rules of differentiation.
9. Over a certain region, the temperature at a point (x, y, z) is given by $T(x, y, z) = 5x^2 - 3xy + xyz$.
   (a) Find the rate of change of the temperature at $(1, 2, 1)$ in the direction of $\langle 1, 1, 1 \rangle$.
   (b) In which direction does T increase most?
   (c) What is the maximum rate of change of T?
10. Find the absolute maximum and minimum values of f on the set D: $f(x, y) = x + y - xy$, where D is the closed triangular region with vertices at $(0, 0), (0, 2)$ and $(4, 0)$.
11. Evaluate by first reversing the order of integration: $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dy$
12. Convert the following integral to polar coordinates (do not evaluate): $\int_0^1 \int_y^{\sqrt{2-y^2}} x + y\, dx\, dy$
13. Given the solid E that is bounded by the cylinder $x^2 + y^2 = 4$, and the planes $z = 0$ and $y + z = 3$:
    (a) Find a parametric representation for the curve of intersection between the cylinder and $y + z = 3$:
    (b) Find a triple integral (and evaluate it) for the volume of E.
14. For each vector field below, estimate as to whether or not it represents a conservative vector field.





15. Set up, but do not evaluate, the surface integral $\iint_S y\,dS$, where S is the surface that consists of $\mathbf{r}(u,v) = \langle uv, u^2, u-2v\rangle$, where $0 \le u \le 1$ and $0 \le v \le 1$.

16. Verify Stokes' Theorem, if $\mathbf{F} = \langle -y, x, -2\rangle$, and the surface S is the cone $z^2 = x^2 + y^2$, $0 \le z \le 4$ oriented downward.

17. Calculate $\iint_S \mathbf{F}\cdot d\mathbf{S}$, if $\mathbf{F} = \langle xy\mathrm{e}^z, xy^2z, -y\mathrm{e}^z\rangle$ and S is the surface of the box bounded by the coordinate planes and $x = 3, y = 2, z = 1$.