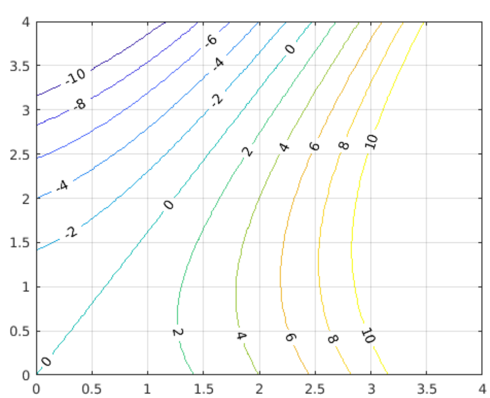
Final Exam Pack C

1. Short Answer/True or False. You may assume that all vectors are in $\mathbb{R}^3$.
   (a) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = 0$.
   (b) $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin(\theta)$
   (c) Given $\mathbf{r}(t)$, then $\mathbf{T}(t) = \mathbf{r}'(t)/|\mathbf{r}'(t)|$ and $\mathbf{N}(t) = \mathbf{T}'(t)/|\mathbf{T}'(t)|$.
2. If $\mathbf{a} \cdot \mathbf{b} = \sqrt{3}$ and $\mathbf{a} \times \mathbf{b} = \langle 1, 2, 2 \rangle$, find the angle between $\mathbf{a}, \mathbf{b}$.
3. Find the limit, if it exists: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4 + y^4}$
4. Find the distance from $(1, 2, 3)$ to the plane $x + 2y - z + 3 = 0$.
5. Find the point at which the line intersects the plane:
$$x = 3 - t, \quad y = 2 + t, \quad z = 5t, \qquad x - y + 2z = 9$$
6. Find the equation of the plane that passes through $(1, 2, 3)$ and contains the line $x = 3t, y = 1 + t, z = 2 - t$.
7. Verify the approximation: $\frac{2x+3}{4y+1} \approx 3 + 2x - 12y$
8. Find the rate of change of f at the point $(0, 2)$ in the direction of $(1, 1)$, if $f(x, y) = ye^{-x}$.
9. Find and classify the critical points of $f(x, y) = xy - 2x - 2y - x^2 - y^2$.
10. Use Lagrange Multipliers to find the maximum of $f(x, y) = 3x + y$ with the constraint $x^2 + y^2 = 10$.
11. Evaluate $\iint_D y^2 e^{xy}\, dA$, where D is bounded by $y = x, y = 4$ and $x = 0$. Be sure to choose the order of integration that will be easier!
12. Let solid E be the solid bounded by paraboloid $y = x^2 + z^2$ and the plane $y = 0$.
   (a) Write two triple integrals that will give the volume, one where the order of integration is $dz\, dy\, dx$, and one where the order is $dy\, dz\, dx$. (Do not evaluate these).
   (b) If we have a vector field $\mathbf{F} = \langle y, z, -x \rangle$, then write an integral that represents the flux of $\mathbf{F}$ across the surface of the paraboloid (you can ignore the plane $y = 0$). (Again, just write the integral, do not evaluate).
13. Given below is the plot of some contours of $f(x, y)$. At the point $(2, 2)$, estimate the sign (positive, negative, or zero) of f_x, f_y, f_{yy} and f_{yx}.



14. Set up, but do not evaluate, the line integral $\int_C xy\,dy$, if the curve C is the bottom half of the unit circle going counterclockwise (CCW).

15. Evaluate $\int_C y^3\,dx - x^3\,dy$ if C is the circle $x^2 + y^2 = 4$.

16. Let C be a simple closed smooth curve that lies in the plane $x + y + z = 1$. Show that the line integral below depends only on the area of the region enclosed by C and not on the shape of C or its location in the plane.

$$\int_C z\,dx - 2x\,dy + 3y\,dz$$

17. Verify the divergence theorem, for the vector field $\mathbf{F} = \langle x, y, z\rangle$ where E is the unit ball $x^2 + y^2 + z^2 \leq 1$.