Exam 3 Review
Math 225

The exam will cover sections 15.1-15.4, 15.7-15.9 (we skipped sections 5, 6, and 10), then also sections 16.1-16.7 (to p. 1114). Lagrange Multipliers (14.8) will not be on the exam.

Double Integrals (15.1-15.4)

Fubini's Theorem, Factoring integrals, Setting up the domain (Type I and II), Use to get the area.

Key skills: General antidifferentiation skills, be able to compute an iterated integral, be able to switch the order of integration. Be able to set up and evaluate an integral using **polar coordinates** (remember the volume element is $r\,dr\,d\theta$).

Triple Integrals (15.6-15.9)

Six setups possible, Cylindrical coordinates, Spherical coordinates (remember the formulas for the volumes!).

Key skills: General antidifferentiation skills, be able to compute an interate integral, be able to switch the order of integration. Be able to set up and evaluate integrals using **cylindrical coordinates** and **spherical coordinates**. Given a sketch (like Figure 1, 15.9), be able to come up with the equations for the spherical coordinates. When integrating, remember the volume elements for the different coordinate systems.

Vector Fields and Line Integrals (16.1-16.4)

- Vocabulary: Vector field, gradient vector field, conservative vector field, potential function.
- Key definitions and notation: Understand what these mean (especially in terms of how to compute them). Note that some of these are alternate notation for the same integral.

$$\int_C f(x,y)\,ds \qquad \int_C f(x,y)\,dx \qquad \int_C f(x,y)\,dy$$

$$\int_C \vec{F}\cdot\vec{T}\,ds \qquad \int_C P\,dx + Q\,dy \qquad \int_C \vec{F}\cdot d\vec{r}$$

- Key skills:
  - Graphical analysis: Match vector fields (plots to equations). Estimate work done by vector field in moving a particle along a given curve. Estimate if a vector field is conservative.
  - Computational skills: Parameterize a curve (especially lines, ellipses, functions of the form $y = f(x)$). Compute the arc length using ds.

- Be able to compute a line integral: Directly, by using the Fundamental Theorem, or by Green's Theorem.
- Understand that the following ideas are all equivalent: Path independence, conservative vector field, line integral on any closed curve is zero, $Q_x - P_y = 0$.
- Two key theorems: The Fundamental Theorem for Line Integrals and Green's Theorem. Recognize that some line integrals represent the area of the enclosed region, like:

$$\oint x\,dy = -\oint y\,dx$$

16.5-16.6: Extensions to Three Dimensions

- Think of **divergence** and **curl** as two new ways of measuring rates of change in a vector field. Divergence was the "spread" of a fluid, and curl was "rotation" of a fluid. Useful vocab: Irrotational and Incompressible (for example, water is an incompressible fluid).

  For two dimensional vector fields, be able to estimate the value of the divergence and curl (as we did in class). Understand why the curl in two dimensions is a scalar, while in three dimensions is a vector.

  - In two dimensions, $\mathbf{F} = \langle P, Q\rangle$
    * Divergence: $P_x + Q_y$
    * Curl: $Q_x - P_y$
  - In three dimensions, $\mathbf{F} = \langle P, Q, R\rangle$.
    * Divergence: $\nabla \cdot \mathbf{F} = P_x + Q_y + R_z$
    * Curl: $\nabla \times \mathbf{F} = (R_y - Q_z)\mathbf{i} + (P_z - R_x)\mathbf{j} + (Q_x - P_y)\mathbf{k}$

- We saw that, in 3-d, "the vector field is conservative" is equivalent to "the curl is zero". We also know that div(curl($\mathbf{F}$) = 0, which gives us a test to see if a given vector field could possibly be the curl of another.

- Useful for later (Can you verify that these are true?)

$$\oint_C \vec{F}\cdot d\vec{r} = \iint_D (\text{curl}(\vec{F}))\cdot\vec{k}\,dA \qquad \oint_C \vec{F}\cdot\vec{n}\,ds = \iint_D \text{div}(\vec{F})\,dA$$

- **Surfaces and Surface Integrals (16.6-7)**
  - Surfaces we consider: $z = f(x, y)$, planes, spheres, cylinders and/or boxes.
  - Formulas for area and volume: The area of a parallelogram defined by vectors $\vec{a}$ and $\vec{b}$ in three dimensions is $|\vec{a}\times\vec{b}|$ (The cross product is only defined for 3-d). The volume of the parallelepiped defined by $\vec{a}, \vec{b}, \vec{c}$ is given by $\vec{a}\cdot(\vec{b}\times\vec{c})$.

- Note that in the special case where $z = g(x, y)$ (where $\mathbf{r}(x, y) = \langle x, y, g(x, y)\rangle$), we have a shortcut formula for the cross product (saves a lot of time!)

$$\mathbf{r}_x \times \mathbf{r}_y = \langle -g_x, -g_y, 1\rangle$$

- The surface area of surface S in two ways- One for $z = g(x, y)$, and one for the generic surface $\mathbf{r}(u, v)$:

$$\iint_S dS = \iint_D \sqrt{g_x^2 + g_y^2 + 1}\, dA \qquad \iint_S dS = \iint_D |\vec{r}_u \times \vec{r}_v|\, dA$$

  Compare these to our two formulas for arc length- one for $y = g(x)$, and the other for the more generic $\mathbf{r}(t)$:

$$\int_C ds = \int_a^b \sqrt{1 + (g'(x))^2}\, dx \qquad \int_C ds = \int_a^b |\mathbf{r}'(t)|\, dt$$

- The surface integral is completely analagous to the line integral using a scalar function. That is, compare the two:

$$\int_C f(x, y)\, ds \qquad \text{vs} \qquad \iint_S f(x, y, z)\, dS$$

We'll stop here (pg 1114) for Exam 3.

Questions to Help with Notation

1. If the curve C is parameterized by $\langle t^2 - t, 2t + 4\rangle$, then compute:
   (a) $ds =$
   (b) $d\vec{r} =$
   (c) Set up the arc length integral, $0 \le t \le 1$.

2. If the surface S is parameterized by $\langle x, y, 3x^2 - xy + 5\rangle$ then compute:
   (a) $\vec{r}_x \times \vec{r}_y =$
   (b) $dS =$
   (c) Write an integral for SA, $if 0 \le x \le 3, -1 \le y \le 2$

3. Is $\int_C \vec{F} \cdot d\vec{r} = \int_C f(x, y)\, ds$? Explain.

4. Is $\int_C \vec{F} \cdot d\vec{r} = \int_C \vec{F} \cdot \vec{T}\, ds$? Explain.

5. Is $\int_C \vec{F} \cdot d\vec{r} = \int_C P\, dx + Q\, dy$? Explain.

6. Is there a difference: $\int_C \vec{F} \cdot d\vec{r}$ versus $\oint_C \vec{F} \cdot d\vec{r}$?

Review Questions (Solutions will be posted to CLEo)

- Chapter 15 Review:
  - True/False: 1-5, 7 and 9.
  - Exercises: 3, 5, 7, 9-10, 12-14, 17-18, 21-22, 26-27, 30-31, 41-42, 47-48.

  Note: I've given you a lot of integrals to compute. I would recommend focusing on getting the integrals *set up* first, and then be sure that you could finish the integration if need be.

- Chapter 16 Review:
  - True/False: All except 9, 11.
  - Exercises: 1-19, 21, 25, 26(a,c), 27-28.
  - Graphical problems: 16.2, p. 1073, # 17-18. 16.3, p. 1082-3, # 1, 11, 25-26. 16.5, p. 1097, # 9-11.

  Same note as before: I've given you a lot of integrals to compute. Really focus on getting your setups correct first, then practice finishing the computations later.