Concepts and Summary: Exam 2, Calculus 3 (Spring 2018)

Sections 13.1-13.4

- Know what a space curve is (in two and three dim). Define a *smooth* curve, and a unit tangent vector.
- Know how to parameterize a line between two points and an off-center ellipse.
- Know how to find a space curve representing the intersection of surfaces.
- Take the limit of a space curve (recall l'Hospital's rule)
- Determine if a space curve is continuous at a point (also recall the definition).
- Given the space curve $\mathbf{r}(t)$, be able to compute its derivative $\mathbf{r}'(t)$, and be able to write the tangent line to a curve at a point.
- Be able to differentiate a sum, the dot product, the cross product of two space curves.
- Differentiate to get velocity and acceleration. Given acceleration, be able to compute velocity and position.
- Be able to integrate a space curve.
- Be able to prove that if $|\mathbf{r}(t)|$ is constant, then $\mathbf{r}' \perp \mathbf{r}$.
- Know the formula for the arc length of a space curve.
- Be able to define the arc length function $s(t)$. In particular, understand that $ds = |\mathbf{r}'(t)|\,dt$
- Know how to compute $\mathbf{T}, \mathbf{N}$ and $\mathbf{B}$ given a space curve $\mathbf{r}(t)$.
- Know what it means to parameterize a function with respect to arc length.
- Know that the definition of curvature is the rate at which the direction on a curve changes: $\kappa = \left|\frac{d\mathbf{T}}{ds}\right|$. I will give you the other formulas for curvature.

Sections 14.1-14.7

- Match a surface with its level curves. Match those with algebraic formulas.
- Know the difference between an explicitly defined surface and an implicitly define surface.

- Be able to sketch level curves in the $xy-$plane. Understand the meaning when the level curves are close together or far apart.

- Be able to find the domain of a given function.

- Be able to tell graphically (given a surface or level curves) if the limit of a function exists or not.

- Be able to show algebraically that a limit does not exist.

- Define continuity of $z = f(x, y)$ at (a, b). Be able to use continuity to prove that a limit exists.

- Be able to use the squeeze theorem to show that a given limit exists.

- Understand why it is much harder to compute a limit for $z = f(x, y)$ in comparison to $y = f(x)$.

- Know the definitions of the partial derivatives and be able to compute a partial derivative using the definition.

- Be able to estimate the partial derivatives given a contour plot (level curves).

- Our definition of differentiable is? (The partial derivatives are each continuous at the point (a, b), this is different than the book)

- Be able to use the subscript and Leibniz notation for the partial derivatives appropriately.

- Know and be able to apply Clairaut's Theorem (mixed partials are equal). In particular, know when it can be applied.

- Be able to compute implicit derivatives. For example, given $f(x, y) = 0$, compute dy/dx. Also, given $f(x, y, z) = 0$, be able to compute z_x and z_y.

- Looked at the relationship between partial derivatives and differentiation, and continuity:

  Just because the partial derivatives exist at a point does not mean that the function is differentiable (or even continuous) at that point.

- f is differentiable if f_x, f_y are continuous at (a, b). This is our definition, which is different than the textbook.

- f is differentiable at (a, b) if it is locally linear at (a, b). Also, if f is differentiable at (a, b), then it is continuous at (a, b).

- The equation of the tangent plane (which is the linearization of f) for $z = f(x, y)$ at (a, b) is:
$$z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$
- For approximations, we can use the total differential:
$$dz = f_x(a, b)dx + f_y(a, b)dy$$
where we take $\Delta x = dx$, $\Delta y = dy$, then use dz to approximate Δz.
- Chain rule in the case that the intermediate functions are functions of one variable, then two variables, then any number of variables. A chart can help you keep track of which is which.

  Technical Note: f should be differentiable, and so should the intermediate variables, for all of our usual formulas to work...
- Implicit differentation in two and three dimensions.
- Two important definitions:
  - The gradient
  - The directional derivative of f at (x_0, y_0) in the direction of unit vector $\mathbf{u} = \langle a, b \rangle$.
- If f is differentiable at (a, b), then $D_u f = \nabla f \cdot \mathbf{u}$.
  - Direction for the maximum rate of change? What is the maximum rate of change?
  - Direction for the minimum rate of change? What is the minimum rate of change?
- Be able to estimate the directional derivative from a contour plot (level curves), and from a numerical table.
- The gradient is orthogonal to the level curve (or surface).
- The tangent plane for implicitly defined functions $F(x, y, z) = k$.
- The normal line at a point on the surface.
- Find (and classify) local extrema for $z = f(x, y)$. We'll need the second derivatives test for this.
- Find (and classify) global (or absolute) extrema. We'll need to evaluate the function at the critical points and along the boundary.