Exam I, Calculus III Review Questions

This review is meant to give you some questions about most of the topics so that you'll have a sense for the level of difficulty and style of the exam questions. The set of questions should not be taken as completely comprehensive, although I have tried to give you a good assortment.

Be sure to look over your old quizzes and homework as well. For more review questions, see the Chapter 12 Review, pg. 834-36.

1. True or False (and give a short reason):

   (a) If the parametric curve $x = f(t), y = g(t)$ satisfies $g'(1) = 0$, then it has a horizontal tangent line when $t = 1$.

   (b) The equations $r = 2$, $x^2+y^2 = 4$, and the set of parametric equations: $(x(t), y(t)) = (2\sin(3t), 2\cos(3t))$ for $0 \leq t \leq 2\pi$, all have the same graph.

   (c) For the next set of questions, assume $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are arbitrary vectors in three dimensions (book notation is V_3, in class we said $\mathbb{R}^3$).

      i. $|\mathbf{u} + \mathbf{v}| = |\mathbf{u}| + |\mathbf{v}|$

      ii. $|\mathbf{u} \cdot \mathbf{v}| = |\mathbf{u}||\mathbf{v}|$

      iii. $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$

      iv. $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$

      v. $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times (\mathbf{v} \times \mathbf{w})$

2. Eliminate the parameter to find a Cartesian equation for the curve: $x = \mathrm{e}^t$, $y = 2\mathrm{e}^t$.

   Find the arc length of the curve from $t = 1$ to $t = 2$ of the original parametric form, then for the (x, y) form.

3. Let $x = t^2 - 2t$ and $y = t + 1$

   (a) At what point does the curve cross the x and y axes?

   (b) Compute dy/dx and d^2y/dx^2.

   (c) Where does the curve have a horizontal tangent line?

   (d) Where does the curve have a vertical tangent line?

   (e) Eliminate the parameter to get x as a function of y.

   (f) Find the area in the *first* quadrant bounded by the curve and the coordinate axes (write your integral in terms of t).

4. A plane is given as $2x + 3y + 4z = 12$. Sketch it in the first octant by drawing the lines of intersection between it and the coordinate planes.

   Give the equations of the lines of intersection that you have drawn (in parametric form).

5. Find the equation of the plane through the points $(3, -1, 1)$, $(4, 0, 2)$ and $(6, 3, 1)$.

6. Convert the polar equation to Cartesian: (i) $r = \tan(\theta)\sec(\theta)$ (ii) $r = 2\cos(\theta)$

7. Find a polar equation for the curve represented by the given Cartesian curve: (i) $y = 1 + 3x$ (ii) $4y^2 = x$

8. Find the slope of the tangent line to the given polar curve at the point specified by $\theta = \pi/3$: $r = 2 - \sin(\theta)$

9. Plot the graph of $r = 2 + 2\sin(\theta)$ in the (r, θ) plane, then transfer that graph to polar coordinates (in the (x, y) coordinate system).

10. Find the distance between the point $(1, 2, 3)$ and the plane $x + y + z = 1$.

11. Find the distance between the planes $x + y + z = 1$ and $3x + 3y + 3z - 5 = 0$.

12. Find the angle between the vectors $\langle 1, 2, 3\rangle$ and $\langle 1, 0, 1\rangle$. Find the projection of the first vector onto the second.

13. A constant force $\mathbf{F} = 3\mathbf{i} + 5\mathbf{j} + 10\mathbf{k}$ moves an object along a line segment from $(1, 0, 2)$ to $(5, 3, 8)$. Find the work done (you may assume the distance is in meters, force is in newtons).

14. Graphical problems: For example #24-27, p 642 (Sect 10.1), #47, 54 on p 663 (Sect 10.3), and #21-28, p 833 (Sect 12.6).

15. Do the lines below intersect? If so, find the point of intersection. If not, find the distance between them.

$$x = 3 + t, y = 2 - t, z = 1 \qquad x = 2 + s, y = 1 + 2s, z = 2 - s$$

16. Write the parametric equations for each situation:

    (a) Go from $(1, 2)$ to $(-3, 2)$ as time runs from 0 to 1.

    (b) Go around the unit circle twice (clockwise) starting at $(-1, 0)$ as $0 \le t \le 1$.

    (c) Parametrize the curve: $y = 3x^2 + 5x + 2$:

    (d) First, plot $r = 1 + 2\cos(\theta)$ in (r, θ) plane, then transfer that plot to a plot in polar coordinates.

17. Find the error (if there is one): Let $\mathbf{u}$ be a vector such that $|\mathbf{u}| = \sqrt{2}$. Choose a vector $\mathbf{v} \neq \mathbf{u}$ such that $\mathbf{u} \cdot \mathbf{v} = 2$. Now we have:

$$\begin{aligned}
2(\mathbf{u}\cdot\mathbf{u}) &= \mathbf{u}\cdot\mathbf{u} + \mathbf{u}\cdot\mathbf{v} \\
2(\mathbf{u}\cdot\mathbf{u}) - 2(\mathbf{u}\cdot\mathbf{v}) &= \mathbf{u}\cdot\mathbf{u} + \mathbf{u}\cdot\mathbf{v} - 2(\mathbf{u}\cdot\mathbf{v}) \\
2(\mathbf{u}\cdot\mathbf{u}) - 2(\mathbf{u}\cdot\mathbf{v}) &= \mathbf{u}\cdot\mathbf{u} - \mathbf{u}\cdot\mathbf{v} \\
2\mathbf{u}\cdot(\mathbf{u} - \mathbf{v}) &= \mathbf{u}\cdot(\mathbf{u} - \mathbf{v}) \\
2\mathbf{u} &= \mathbf{u} \\
2 &= 1
\end{aligned}$$

18. Convert $r = 2\cos(\theta)$ to Cartesian coordinates, and write the equation of the circle in standard form.

19. Find the scalar and vector projections of $\mathbf{a}$ onto $\mathbf{b}$, if

    - $\mathbf{a} = \langle -2, 3, -6\rangle \qquad \mathbf{b} = \langle 5, -1, 4\rangle$
    - $\mathbf{a} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{b} = \mathbf{j} - \mathbf{k}$

20. Sketch a few traces of the surface $y^2 + z^2 = 1 + x^2$ and describe the resulting surface (in words and/or as a sketch in three-dimensions).

21. Let a line L and a plane P be defined as:

$$L: \quad x = \frac{y+2}{3} = \frac{z}{-1} \qquad P_1: \quad x + y + z = 1$$

    (a) Write the equation of the line in parametric form. In what follows, you may feel free to use either equation.
    (b) Find the point of intersection between the plane and the line.
    (c) Find the plane (P_2) that contains the point $Q(1, 2, 1)$, and line L.
    (d) Find the distance between point Q and the plane P_1. What is the distance between planes P_1, P_2?
    (e) If planes P_1, P_2 intersect, find the line of intersection.

22. Is this possible? (Give a short reason)

    Let $\mathbf{u}$ be a vector such that $|\mathbf{u}| = 1$. Choose a vector $\mathbf{v}$ such that $\mathbf{u} \cdot \mathbf{v} = 3$ and $|\mathbf{v}| = \sqrt{5}$.

23. Find the volume of the parallelepiped determined by the vectors: $\mathbf{i} + \mathbf{j}$, $\mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} + \mathbf{k}$.

24. Write the vector $\langle -1, 5, 3\rangle$ as a sum of a vector *parallel* to $\langle 4, 2, 4\rangle$ and a vector *perpendicular* to $\langle 4, 2, 4\rangle$ (Hint: Think projection, and perhaps draw a quick sketch).