Selected Exercise Solutions, 14.3

For exercise 4, we have the table (duration, t, in hours are horizontal, wind speed v, in knots, along the vertical):

	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

- Meaning of $f_v(v,t) = \frac{\partial h}{\partial v}$?

  Describes how quickly the wave heights change for a fixed time duration.

- Meaning of $f_t(v,t)$?

  Represents how quickly the wave heights change when the duration of time changes, but the wind speed is constant.

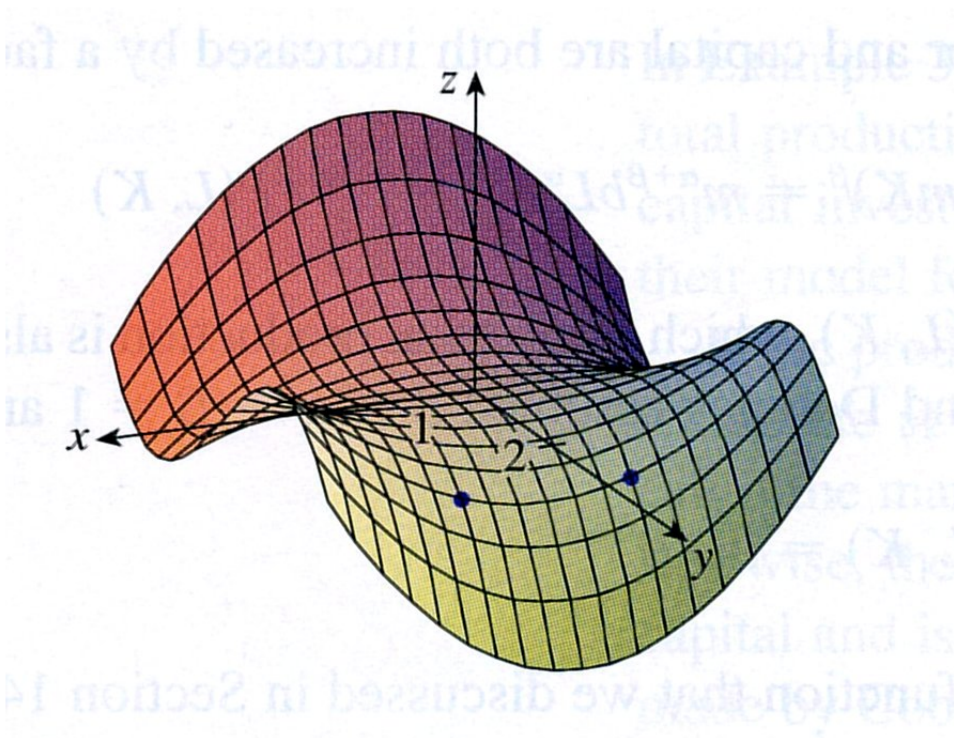
- $$f_v(40,15) \approx \frac{36-25}{10} \approx 1.1 \qquad \text{or} \quad \frac{16-25}{-10} = 0.9$$

  Average them to get approximately 1.

- What appears to be:

  $$\lim_{t\to\infty} f_t(v,t)?$$

  SOLUTION: For any fixed velocity, $f(v,t)$ becomes more and more constant over time. Therefore, $f_t(v,t) \to 0$.



For Exercises 5-8, here is some of the reasoning we were looking for:

5. $f_x(1,2) > 0$ and $f_y(1,2) < 0$

6. $f_x(-1,2)$ and $f_y(-1,2)$

   The graph of f decreases if we start at $(-1,2)$ and move in the positive $x-$direction, so $f_x(-1,2) < 0$. Similarly, $f_y(-1,2) < 0$.

7. $f_{xx}(-1,2)$ and $f_{yy}(-1,2)$

   f_xx is the rate of change of f_x as we move in the positive $x-$direction. Starting at $(-1,2)$, f_x is negative the surface becomes less steep by moving in the positive $x-$direction. Therefore, f_x is INCREASING, and $f_{xx}(-1,2) > 0$.

   Similarly, moving in the positive y direction, f_y begins negative, and because the surface becomes more steep, f_y becomes more negative. Therefore, f_y is DECREASING and $f_{yy}(-1,2) < 0$.

8. $f_{xy}(1,2)$

   We think of f_{xy} as the rate of change of f_x in the y direction. f_x is positive at $(1,2)$. Shifting slightly forward in y, we see that the surface becomes more steep (looking in x direction), so f_x will increase. Therefore, $f_{xy}(1,2) > 0$.

   Convince yourself that $f_{xy}(-1,2) < 0$ by seeing that f_x is negative and would get more negative.