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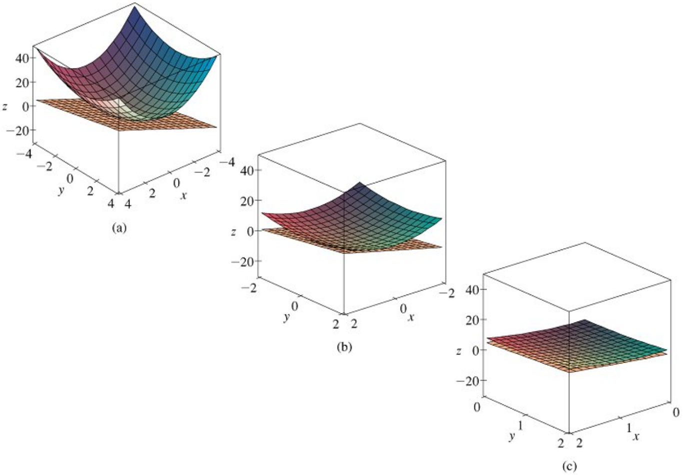
- If f is differentiable at $x = a$, then it is continuous at $x = a$ (The other way doesn't work- $y = |x|$ at $x = 0$).

What Should Differentiability Be?

A function $z = f(x, y)$ should be "differentiable" at (a, b) if it is locally linear there.


40
20
z 0
−20
−4
−2
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y
2
4 4
2
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−2
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(a)
40
20
z 0
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−2
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2 2
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20
z 0
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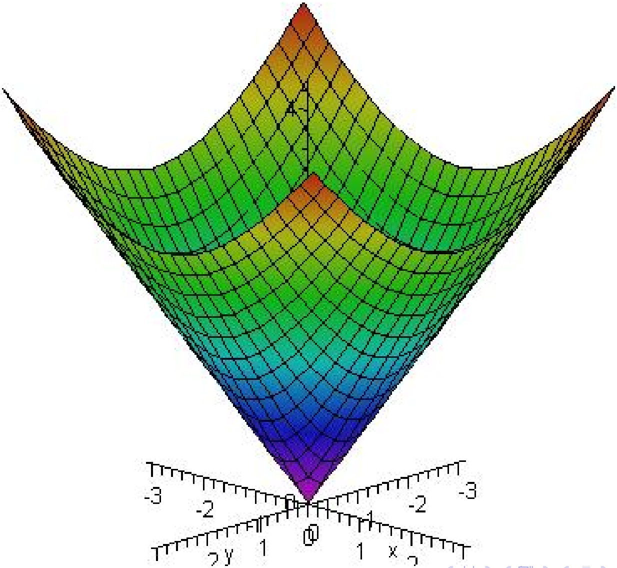
y
2 2
1
x
0
(c)


-3
-2
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If we keep in mind that f should be locally linear in order to be differentiable, then consider the following examples:

Example 1

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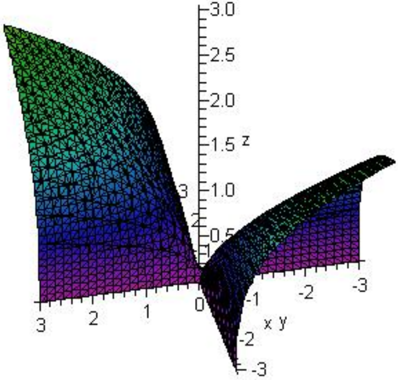
The partial derivatives do not exist at the origin, and this function is not differentiable at the origin.

Example 2

$$f(x, y) = x^{1/3}y^{1/3}$$



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Similarly, $f_y(0,0) = 0$ by using the definition.
The function may fail to be differentiable at a point, even though the partial derivatives exist at the point.

Example 3:

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Show that $f_x(0,0)$ and $f_y(0,0)$ both exist:

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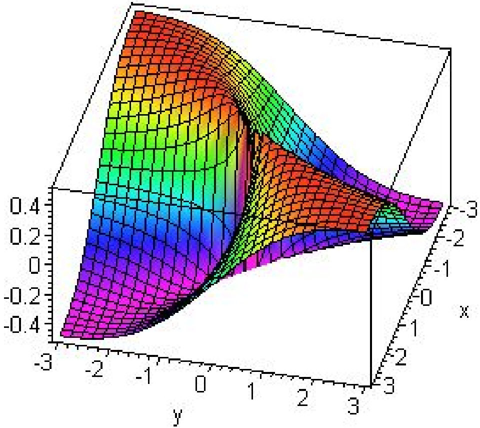
Similarly, $f_y(0,0) = 0$.
This function is NOT continuous at the origin (consider $y = x$ and $y = -x$).
The partial derivatives may exist, even though the function is not continuous at a point. (In this case, f is not differentiable, either).


0.4
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Differentiability Theorem

If the partial derivatives exist and are continuous on a small disk centered at (a, b), then $z = f(x, y)$ is differentiable at (a, b).