Exam 1 Sample Questions

Be sure to look over your old quizzes and homework as well.

1. Eliminate the parameter to find a Cartesian equation for the curve: $x = e^t$, $y = 2e^t$.

   Find the arc length of the curve from $t = 1$ to $t = 2$ of the original parametric form, then for the (x, y) form.

2. Find dy/dx and d^2y/dx^2, if $x = t^3 - 12t$ and $y = t^2 - 1$ (Hint: We do NOT want to try to convert it first).

3. Convert the polar equation to Cartesian:

$$r = \tan(\theta)\sec(\theta)$$

4. Convert the equation from Cartesian to polar of the form $r = f(\theta)$.

$$xy = 4$$

5. Find the area of the surface obtained by rotating the curve about the $x-$axis:

$$x = 3t - t^3 \qquad y = 3t^2 \qquad 0 \le t \le 1$$

6. Find the slope of the tangent line to the given polar curve at the point specified by θ:

$$r = 2 - \sin(\theta) \qquad \theta = \frac{\pi}{3}$$

7. Show that the equation $r = a\sin(\theta) + b\cos(\theta)$, where $ab \neq 0$, represents the equation of a circle.

8. If we have two parallel planes, P_1 and P_2:

$$P_1: \; ax + by + cz + d_1 = 0 \qquad P_2: \; ax + by + cz + d_2 = 0$$

   Then show that the distance between the planes is

$$\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

9. Graphical problems: Like the back of Quiz 3 (online), prob. 24, 25, 26 on p. 627.

10. Do the lines below intersect? If so, find the point of intersection. If not, find the distance between them.

$$x = 3 + t, y = 2 - t, z = 1 \qquad x = 2 + s, y = 1 + 2s, z = 2 - s$$

11. Find an equation for the surface obtained by rotating the parabola $y = x^2$ about the $y-$axis. (Hint: In 3-d, if you fix a $y-$value, what shape should you have in the $xz-$plane?)

12. Find the distance between the origin and the line

$$x = 1 + t \quad y = 2 - t \quad z = -1 + 2t$$

Hint: Take an arbitrary point on the line and form two vectors so that the distance can be found- perhaps with a sine? Hint 2: It is possible to solve this with Calculus- Use it to check your answer.

13. Given the property:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

If vectors $\mathbf{a}$ and $\mathbf{b}$ are unit vectors, and

$$\mathbf{c} = \mathbf{a} \times (\mathbf{a} \times \mathbf{b})$$

- Is $\mathbf{c}$ a unit vector?
- Is $\mathbf{c} \perp \mathbf{a}$?
- Is $\mathbf{c} \perp \mathbf{b}$?

14. Assume that $\mathbf{a} \neq \mathbf{0}$. Explain your answer (if "No", provide an example):

(a) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?

(b) If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?

(c) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ and $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?