Selected Solutions, Chapter 15 Review

T/F, 7 False. Notice that the r is missing: It should be $r\,dz\,dr\,d\theta$

10. $\displaystyle\int_0^4 \int_{y-4}^{4-y} f(x,y)\,dx\,dy$

12. The region is in the first octant on or between the spheres of radius 1 and radius 2.

14. $\displaystyle\int_0^1 \int_0^{x^2} \frac{y\mathrm{e}^{x^2}}{x^3}\,dy\,dx = \frac{1}{4}(\mathrm{e}-1)$

18. Best to integrate in y first (otherwise you have to add two integrals together):

$$\int_0^1 \int_x^1 \frac{1}{1+x^2}\,dy\,dx = \int_0^1 \frac{1}{1+x^2} - \frac{x}{1+x^2}\,dx$$

The first antiderivative is $\tan^{-1}(x)$. To do the second, we can use $u = 1+x^2$ and $du = 2x\,dx$. Numerically, we get:

$$\tan^{-1}(1) - \tan^{-1}(0) - \frac{1}{2}\ln(2) + \frac{1}{2}\ln(1) = \frac{\pi}{4} - \frac{1}{2}\ln(2)$$

20. $\displaystyle\int_1^2 \int_{1/y}^{y} y\,dx\,dy = \frac{4}{3}$

26. (See figure) $\displaystyle\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{2-y} z\,dx\,dz\,dy = \frac{13}{24}$

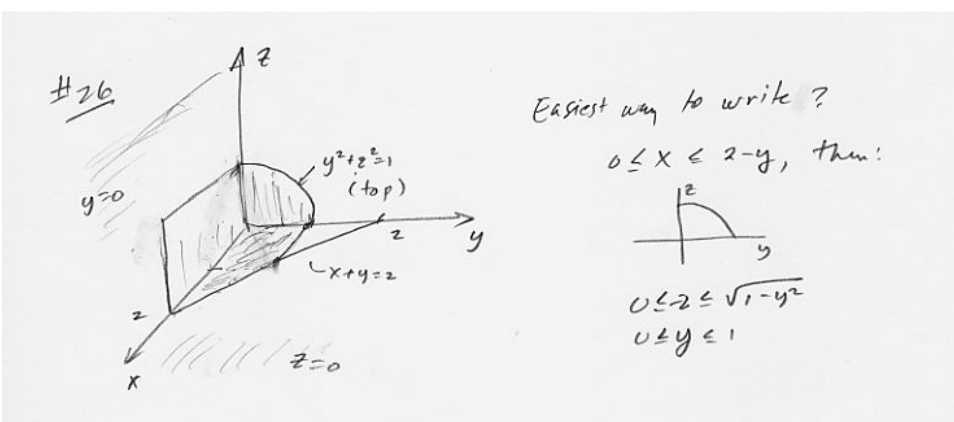

Figure 1: Figure for Exercise 26, Chapter 15 Review

30. $\displaystyle\int_0^1 \int_{y+1}^{4-2y} \int_0^{x^2 y} dz\,dx\,dy = \frac{53}{20}$

46. See the figure below.

Selected Solutions, Chapter 16 Review

True/False (2) True, (4) True, (6) False (the arc length is independent of the parameterization), (8) False, since the divergence of the curl is not zero.

2. $\frac{1}{12}(5\sqrt{5}-1)$

4. Conservative vector field, so 0.

   Side Remark: If you were curious, to get the parameterization re-write the ellipse in standard form:

$$\frac{x^2}{9}+\frac{y^2}{4}=1 \quad \Rightarrow \quad x(t)=3\cos(t) \quad y(t)=2\sin(t)$$

6. $\mathrm{e}-\frac{9}{70}$

8. $\pi/4$

10. In part (a), parameterize as $x=3-3t$, $y=\frac{\pi}{2}t$, $z=3t$ for $0\le t\le 1$. With that, get $\frac{1}{2}(3\pi-9)$.

    In part (b), you should get $-3\pi/4$ (notice that this vector field is not path independent).

14. 2

16. 3

18. Messy, but straightforward:

$$\langle -\mathrm{e}^{-y}\cos(z), -\mathrm{e}^{-z}\cos(x), -\mathrm{e}^{-x}\cos(y)\rangle$$

    The divergence does not simplify too nicely, either:

$$-\mathrm{e}^{-x}\sin(y)-\mathrm{e}^{-y}\sin(z)-\mathrm{e}^{-z}\sin(x)$$

38. **Deleted**

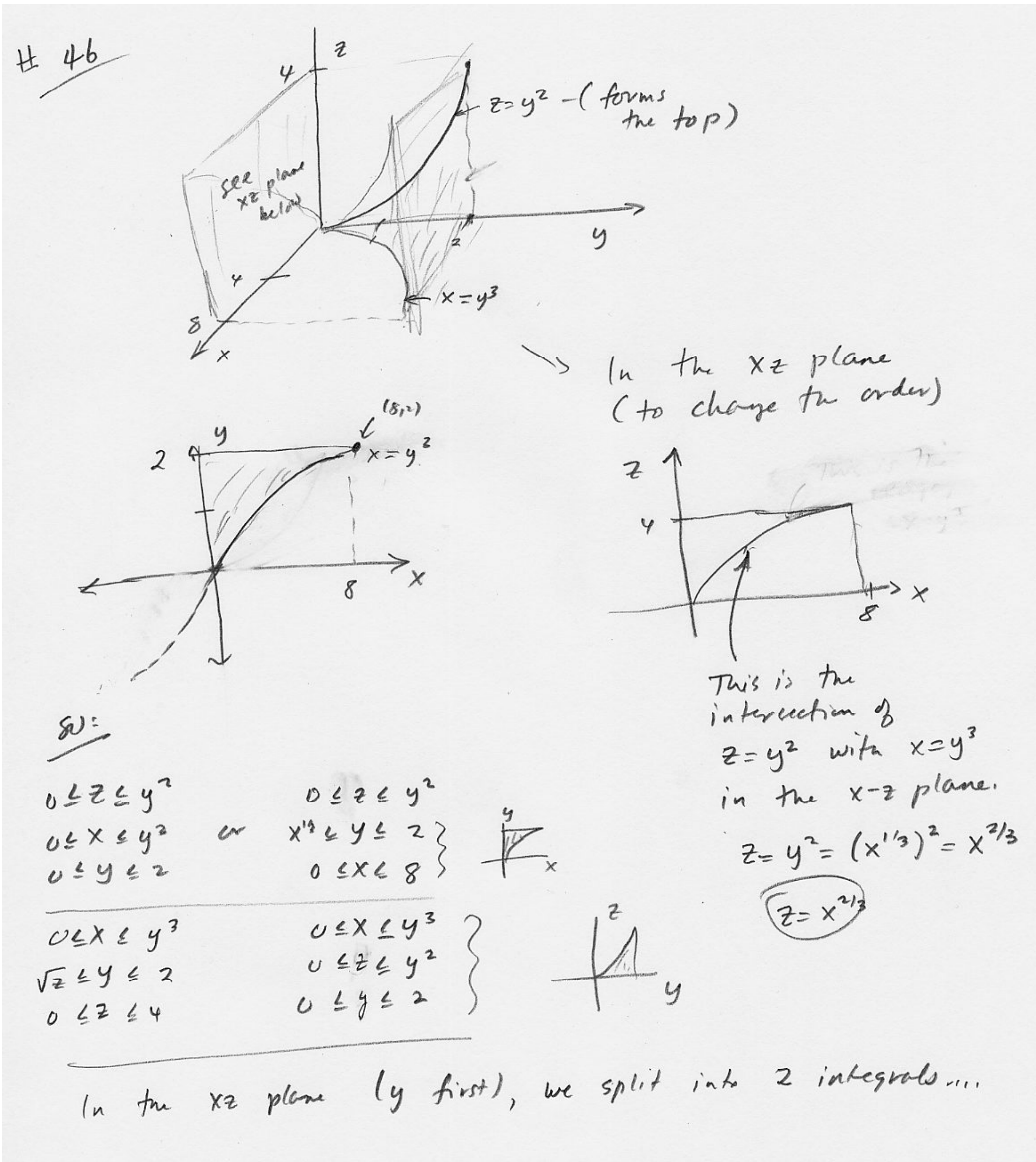

Figure 2: Figure for Exercise 46, Chapter 15 Review