Exam 1: Sections 10.1-12.6

Definitions

Parametric equations, polar coordinates. Coordinate planes, equations of lines, planes and spheres. Vectors (and standard basis vectors $\vec{i}, \vec{j}, \vec{k}$). Unit vector. Dot product, Cross product. Parallel vectors, parallel planes.

Remark: We did not say it explicitly in class, but you should note that the cross product is special in the sense that it is defined ONLY for vectors in three dimensions. On the other hand, a dot product can be defined for vectors with any number of elements.

Computations

- Slope of tangent line (for parametric equations), Area of curve (given in parametric form), arc length (for curve in parametric form), surface area for an object found by rotating a curve about an axis.
- Polar coordinate to Cartesian coordinate conversion (and vice-versa). Slope of tangent line to polar curve.
- Distance formula in 3-d, Equation of a sphere.
- Operations on vectors: Add, subtract, scalar multiplication, magnitude. Construct a vector (head minus tail) Construct a unit vector.
- Find the equation of line/plane. Recall there are multiple ways of expressing a line or a plane- Important: think about what is needed to define each, then how you convert to the different forms.
- Compute the distance between: a point and a line (2d), a point and a plane (3d), two planes, two (skew) lines.
- Points of intersection between lines; line of intersection between planes, point of intersection between a line and a plane.
- For 12.6, be able to sketch ellipses and hyperboli in two dimensions, then translate to 3d. I will not ask you to memorize the names or the forms of the surfaces in the table on p. 808 (although you can impress your friends that way!).

Theory

- There are two ways of computing the dot product:
- There are two ways of computing the cross product:
- Depending on the which lengths of a triangle is desired, we have:

$$\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|} \qquad \sin(\theta) = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|\,|\mathbf{b}|}$$

- Important properties of the dot product: $|\vec{a}|^2 = \vec{a} \cdot \vec{a}$, dot product is symmetric, dot product used for orthogonality and projection. The dot product is used to compute Work.

- Important properties of the cross product: It produces a vector that is orthogonal to the given vectors, Geometrically, its magnitude is the area of the parallelogram, can be used to see if vectors are parallel, Volume of a parallelepiped (shortcut for computation).

Some more sample questions to be posted later.