34. The dimensions of a closed rectangular box are measured as 80 cm, 60 cm, and 50 cm, with a possible error of 0.2 cm in each dimension. Use differentials to estimate the maximum error in calculating the surface area of the box.

**SOLUTION:** Let the dimensions of the box be $l, w$ and $h$ (for length, width and height). The surface area is then:

$$ S(l, w, h) = 2lw + 2wh + 2lh = 2(lw + wh + lh) $$

The change in area can be written as:

$$ \Delta S \approx dS = S_l \, dl + S_w \, dw + S_h \, dh $$

where the partial derivatives are evaluated at $l = 80$, $w = 60$ and $h = 50$, and $dl = dw = dh = 0.2$.

The partial derivatives are computed:

$$ S_l = 2(w + h) = 220 \quad S_w = 2(l + h) = 260 \quad S_h = 2(l + w) = 280 $$

Substituting these in for $dS$,

$$ dS = 220 \cdot 0.2 + 260 \cdot 0.2 + 280 \cdot 0.2 = 152 \text{ cm}^2 $$

35. Use differentials to estimate the amount of tin in a closed tin can with diameter 8 cm and height 12 cm if the tin is 0.04 cm thick.

**SOLUTION:** The volume of the can is

$$ V(r, h) = \pi r^2 h $$

Using differentials,

$$ \Delta V \approx dV = V_r \, dr + V_h \, dh $$

with $r = 4$ and $h = 12$, $dr = 0.04$ and $dh = 2 \cdot 0.04 = 0.08$.

Compute the partial derivatives out and we get:

$$ dV = 2\pi rh \, dr + \pi r^2 \, dh $$

Substitute in the numerical values:

$$ dV = 3.84\pi + 1.28\pi \approx 16.08 \text{ cm}^3 $$

(The book rounds off a little too much)
36. Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 cm high and 4 cm in diameter if the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick.

SOLUTION: Same volume equation as before \((V = \pi r^2 h)\):

\[
dV = 2\pi rh \, dr + \pi r^2 h \, dh
\]

In this case, \(r = 2\), \(h = 10\), \(dr = 0.05\) and \(dh = 2 \cdot 0.1 = 0.2\). Substituting these values in, we get:

\[
dV = 2.80\pi \approx 8.8 \text{ cm}^3
\]

37. A boundary stripe is 3 inches wide and is painted around a rectangle whose dimensions are 100 \(\times\) 200 feet. Use differentials to approximate the number of square feet of paint in the stripe.

SOLUTION: Use the area: \(A = xy\), so that

\[
\Delta A \approx dA = A_x \, dx + A_y \, dy = y 
\]

Notice that the change in \(x\) will be twice the width of a stripe (in feet), so

\[
dx = dy = 6 \text{ in} = 1/2 \text{ ft}
\]

Substituting these in,

\[
dA = 100 \cdot \frac{1}{2} + 200 \cdot \frac{1}{2} = 150 \text{ ft}^2
\]

38. Given that pressure, volume and temperature of a gas are related by:

\[
PV = 8.31T
\]

use differentials to find the approximate change in pressure if the volume increases from 12L to 12.3L and temperature decreases from 310 Kelvin to 305 Kelvin.

SOLUTION: A couple of ways to go about this. Easiest way might be to first write \(P\) as a function of \(T, V\) since we want to approximate \(\Delta P:\)

\[
P = 8.31 \frac{T}{V} \Rightarrow dP = \frac{8.31}{V} \, dT - \frac{8.31T}{V^2} \, dV
\]

with \(T = 310, \, dT = -5, \, V = 12\) and \(dV = 0.3\). Substitute these in:

\[
dP = \frac{8.31}{12} \cdot (-5) - \frac{8.31 \cdot 310}{12^2} \cdot 0.3 \approx -8.83
\]
41. A model for the surface area of a human body is given by

\[ S = 0.109w^{0.425}h^{0.725} \]

where \( w \) is weight (in pounds), \( h \) is in feet, and \( S \) in square feet. If the errors in measuring \( w \) and \( h \) are at most 2%, use differentials to estimate the maximum percentage error in calculating S.A.

SOLUTION: The percentage error is given by:

\[ \frac{\Delta S}{S} \approx \frac{dS}{S} = \frac{S_w \, dw + S_h \, dh}{S} \]

We are also given: \( dw = 0.02w \) and \( dh = 0.02h \). Now,

\[ \frac{S_w}{S} \, dw = \frac{(0.109 \cdot 0.425)w^{0.425-1}h(0.02)w}{S} = 0.425 \cdot 0.02 \]

Similarly,

\[ \frac{S_h}{S} \, dh = \frac{0.109 \cdot 0.725w^{0.425}h^{0.725-1}0.02h}{S} = 0.725 \cdot 0.02 \]

Add them together to get the desired result: 0.023, or about 2.3%.

For fun, did you see how much surface area you have?