SOLUTIONS: Section 14.4, "Story Problems"

34. The dimensions of a closed rectangular box are measured as 80 cm, 60 cm, and 50 cm, with a possible error or 0.2 cm in each dimension. Use differentials to estimate the maximum error in calculating the surface area of the box.

    SOLUTION: Let the dimensions of the box be l, w and h (for length, width and height). The surface area is then:

    $$S(l, w, h) = 2lw + 2wh + 2lh = 2(lw + wh + lh)$$

    The change in area can be written as:

    $$\Delta S \approx dS = S_l\, dl + S_w\, dw + S_h\, dh$$

    where the partial derivatives are evaluated at $l = 80$, $w = 60$ and $h = 50$, and $dl = dw = dh = 0.2$.

    The partial derivatives are computed:

    $$S_l = 2(w + h) = 220 \qquad S_w = 2(l + h) = 260 \qquad S_h = 2(l + w) = 280$$

    Substituting these in for dS,

    $$dS = 220 \cdot 0.2 + 260 \cdot 0.2 + 280 \cdot 0.2 = 152 \text{ cm}^2$$

35. Use differentials to estimate the amount of tin in a closed tin can with diameter 8 cm and height 12 cm if the tin is 0.04 cm thick.

    SOLUTION: The volume of the can is

    $$V(r, h) = \pi r^2 h$$

    Using differentials,

    $$\Delta V \approx dV = V_r\, dr + V_h\, dh$$

    with $r = 4$ and $h = 12$, $dr = 0.04$ and $dh = 2 \cdot 0.04 = 0.08$.

    Compute the partial derivatives out and we get:

    $$dV = 2\pi rh\, dr + \pi r^2\, dh$$

    Substitute in the numerical values:

    $$dV = 3.84\pi + 1.28\pi \approx 16.08 \text{ cm}^3$$

    (The book rounds off a little too much)

36. Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 cm high and 4 cm in diameter if the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick.

SOLUTION: Same volume equation as before ($V = \pi r^2 h$):

$$dV = 2\pi r h\, dr + \pi r^2\, h\, dh$$

In this case, $r = 2$, $h = 10$, $dr = 0.05$ and $dh = 2 \cdot 0.1 = 0.2$. Substituting these values in, we get:

$$dV = 2.80\pi \approx 8.8 \text{ cm}^3$$

37. A boundary stripe is 3 inches wide and is painted around a rectangle whose dimensions are 100×200 feet. Use differentials to approximate the number of square feet of paint in the stripe.

SOLUTION: Use the area: $A = xy$, so that

$$\Delta A \approx dA = A_x\, dx + A_y\, dy = y\, dx + x\, dy$$

Notice that the change in x will be twice the width of a stripe (in feet), so

$$dx = dy = 6 \text{ in} = 1/2 \text{ ft}$$

Substituting these in,

$$dA = 100 \cdot \frac{1}{2} + 200 \cdot \frac{1}{2} = 150 \text{ ft}^2$$

38. Given that pressure, volume and temperature of a gas are related by:

$$PV = 8.31T$$

use differentials to find the approximate change in pressure if the volume increases from 12L to 12.3L and temperature decreases from 310 Kelvin to 305 Kelvin.

SOLUTION: A couple of ways to go about this. Easiest way might be to first write P as a function of T, V since we want to approximate ΔP:

$$P = 8.31\frac{T}{V} \quad \Rightarrow \quad dP = \frac{8.31}{V}\, dT - \frac{8.31T}{V^2}\, dV$$

with $T = 310$, $dT = -5$, $V = 12$ and $dV = 0.3$. Substitute these in:

$$dP = \frac{8.31}{12} \cdot (-5) - \frac{8.31 \cdot 310}{12^2} \cdot 0.3 \approx -8.83$$

41. A model for the surface area of a human body is given by

$$S = 0.109w^{0.425}h^{0.725}$$

where w is weight (in pounds), h is in feet, and S in square feet. If the errors in measuring w and h are at most 2%, use differentials to estimate the maximum percentage error in calculating S.A.

SOLUTION: The percentage error is given by:

$$\frac{\Delta S}{S} \approx \frac{dS}{S} = \frac{S_w\,dw + S_h\,dh}{S}$$

We are also given: $dw = 0.02w$ and $dh = 0.02h$. Now,

$$\frac{S_w}{S}\,dw = \frac{(0.109 \cdot 0.425)w^{0.425-1}h(0.02)w}{S} = 0.425 \cdot 0.02$$

Similarly,

$$\frac{S_h}{S}\,dh = \frac{0.109 \cdot 0.725w^{0.425}h^{0.725-1}0.02h}{S} = 0.725 \cdot 0.02$$

Add them together to get the desired result: 0.023, or about 2.3%.

For fun, did you see how much surface area you have?