Chapter 14

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Definition

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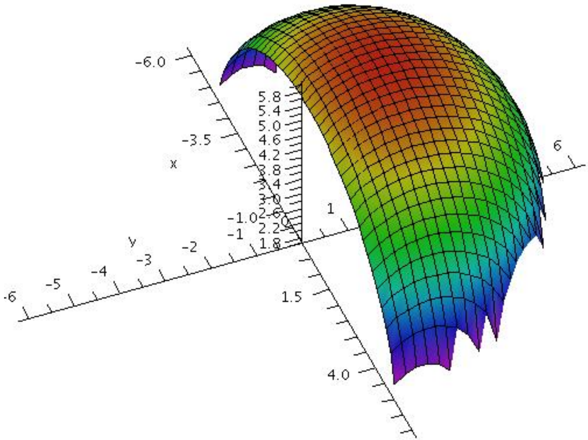
$$\sqrt{y} \Rightarrow y \geq 0$$

$$25 - x^2 - y^2 \geq 0 \Rightarrow x^2 + y^2 \leq 25$$


-6.0
-3.5
x
-1.0
1.5
4.0
5.8
5.4
5.0
4.6
4.2
3.8
3.4
3.0
2.6
2.2
1.8
y
-6
-5
-4
-3
-2
-1
1
6

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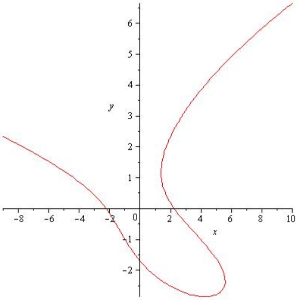
Example: $\quad x^2 + 3xy - y^3 = 5$



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We might think of time as one of the dimensions...

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We might think of time as one of the dimensions...

To visualize three-dimensional graphs, look at the "level curves":

Definition

The **level curves** for a function $z = f(x, y)$ are curves where $k = f(x, y)$ (and note these are curves in the plane).

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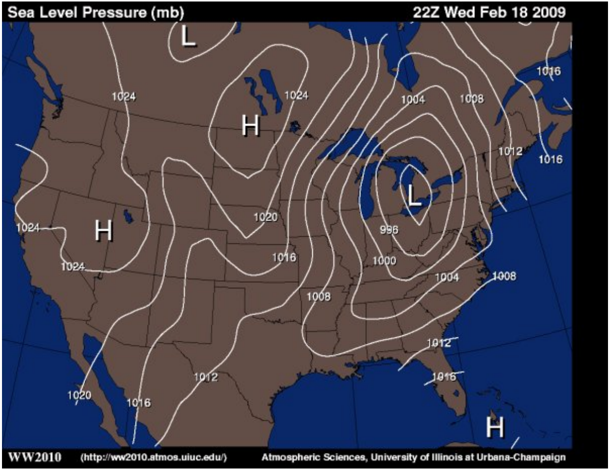
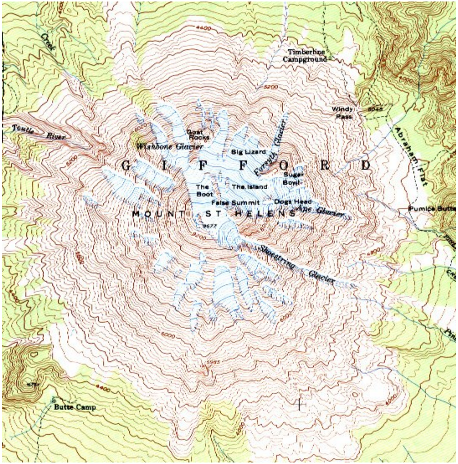
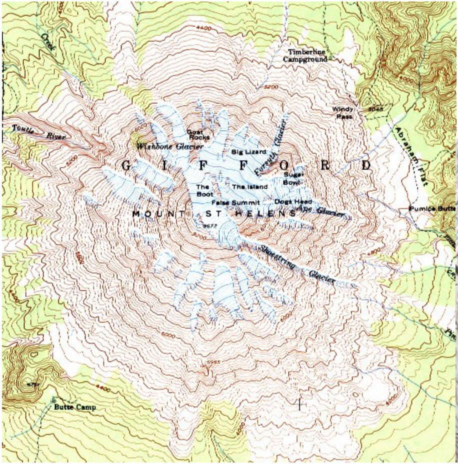



Timberline Campground
Windy Pass
Abraham Flat
Pumice Butte
Toutle River
Creek
Goat Rocks
Wishbone Glacier
Big Lizard
Forsyth Glacier
G I F F O R D
Sugar Bowl
The Boot
The Island
False Summit
Dogs Head
M O U N T S T H E L E N S
Shoestring Glacier
Butte Camp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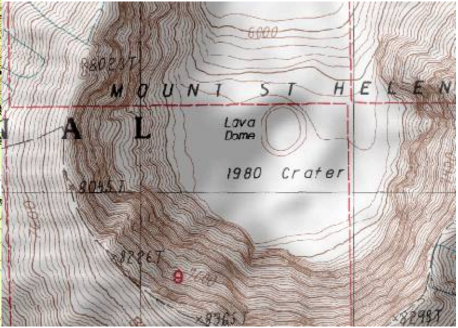
MOUNT ST HELEN
A L
Lava Dome
1980 Crater

Reading Level Curves

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Level curves are packed together.

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Shallow surface
Level curves are far apart.

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Now a mathematical example:

Example: Plot the function $(x^2 + 3y^2)\mathrm{e}^{-x^2-y^2}$ by looking at it in 3-d and with the level curves: