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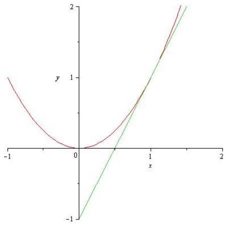

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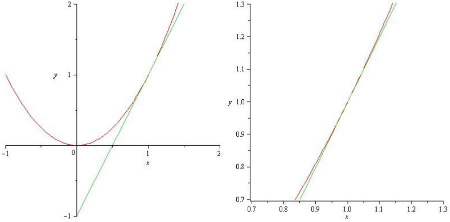

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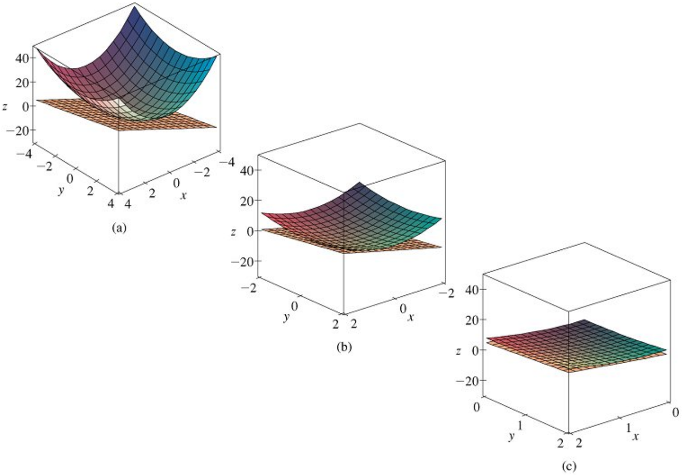
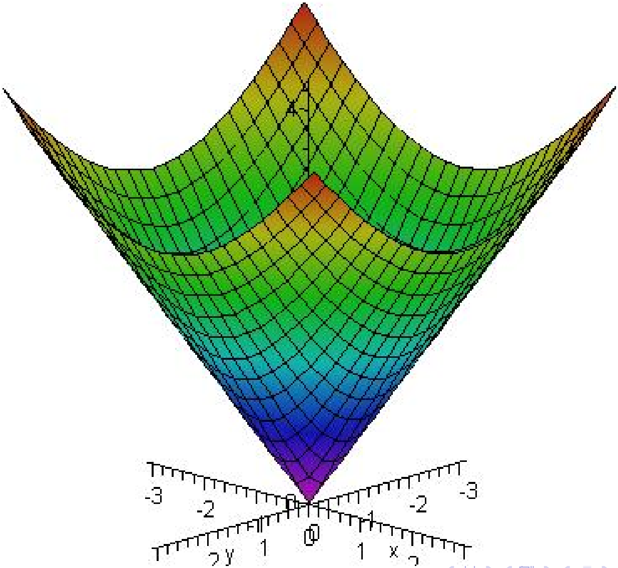
This *should* guarantee the existence of the partial derivatives and the continuity of $z = f(x, y)$ at a point (a, b).



40
20
z 0
−20
−4
−2
0
y
2
4 4
2
0 x
−2
−4
(a)
40
20
z 0
−20
−2
0
y
2 2
0 x
−2
(b)
40
20
z 0
−20
0
1
y
2 2
1 x
0
(c)



-3
-2
-3
-2
y
1
0
1
x

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"Differentiable" is a stronger condition than existence of the partial derivatives,
But if the partial derivatives are **continuous** at (a, b), then f is differentiable there (in the sense of being locally linear).

Example:

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

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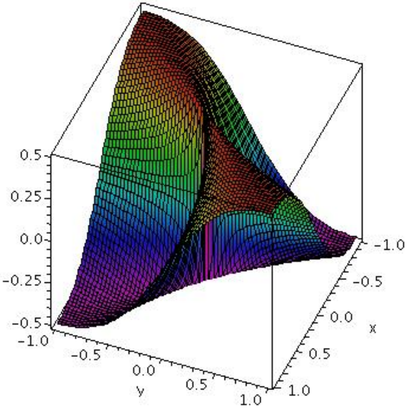

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This function is NOT continuous at the origin (consider $y = x$ and $y = -x$).
The partial derivatives may exist, even though the function is not continuous at a point.

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The **definition** of differentiability is somewhat complicated. We will use the following theorem instead:

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Differentiability Theorem

If the partial derivatives exist and are continuous on a small disk centered at (a, b), then $z = f(x, y)$ is differentiable at (a, b).

If $z = f(x, y)$ is differentiable at (a, b), then we can use the tangent plane to approximate it. That is, either directly:

$$z - f(a, b) = f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

Or indirectly: Let $dx = \Delta x = x - a$ and $dy = \Delta y = y - b$. Then the **total differential** dz is approximately Δz,

$$\Delta z \approx dz = f_x(a, b)\, dx + f_y(a, b)\, dy$$

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Therefore, using $f(7,2) = \ln(1) = 0$, we have:

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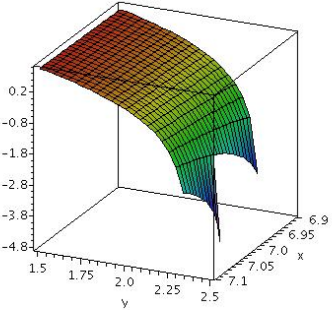
$$f(6.9, 2.06) \approx 0 + 1 \cdot (-0.1) - 3(0.06) = -0.28$$


0.2
-0.8
-1.8
-2.8
-3.8
-4.8
1.5
1.75
2.0
2.25
2.5
y
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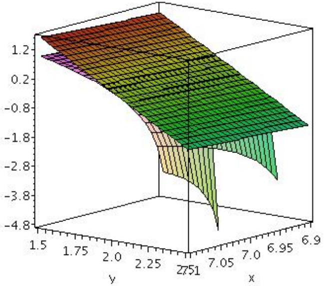
6.95
7.0
7.05
7.1
x


1.2
0.2
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