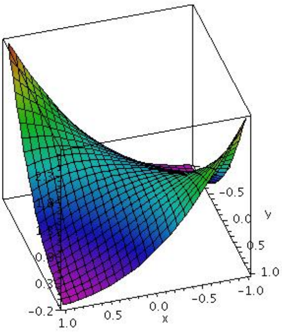
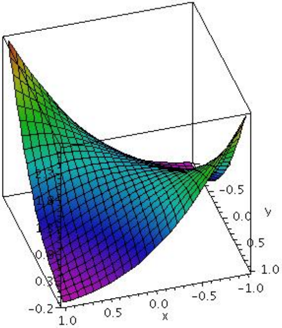


Does the limit exist at the origin?

$$f(x, y) = \mathrm{e}^{-xy}\cos(x + y)$$

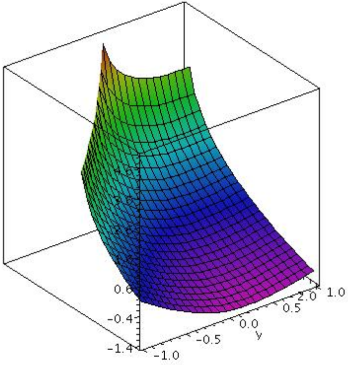


Does the limit exist at the origin?

$$f(x,y) = \mathrm{e}^{-xy}\cos(x+y)$$

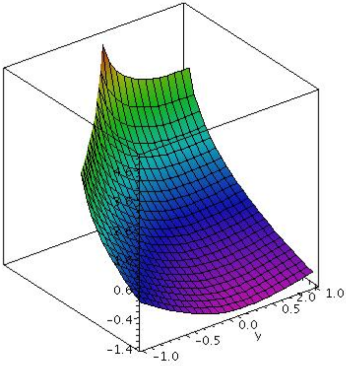
SOLUTION:

f is continuous at the origin.
The limit is $f(0,0) = 1$.



Does the limit exist at $(1, 0)$?

$$f(x, y) = \ln\left(\frac{1 + y^2}{x^2 + xy}\right)$$



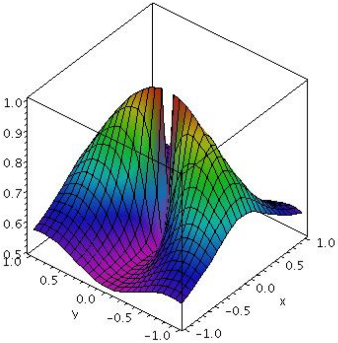
Does the limit exist at $(1,0)$?

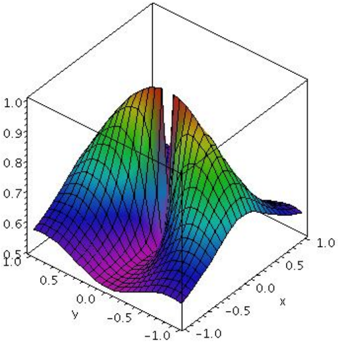
$$f(x,y) = \ln\left(\frac{1+y^2}{x^2+xy}\right)$$

SOLUTION:
f is continuous at $(1,0)$. The limit is $f(1,0) = \ln(1) = 0$.

Does the limit exist at $(0, 0)$?

$$f(x, y) = \frac{x^2 + \sin^2(y)}{2x^2 + y^2}$$





Does the limit exist at $(0, 0)$?

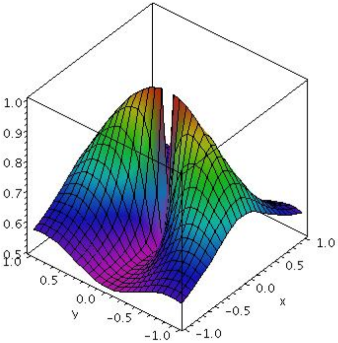
$$f(x, y) = \frac{x^2 + \sin^2(y)}{2x^2 + y^2}$$

SOLUTION:
Limit along $x = 0$:

$$\lim_{(x,y)\to(0,0)} \left(\frac{\sin(y)}{y}\right)^2 =$$

Does the limit exist at $(0, 0)$?

$$f(x, y) = \frac{x^2 + \sin^2(y)}{2x^2 + y^2}$$

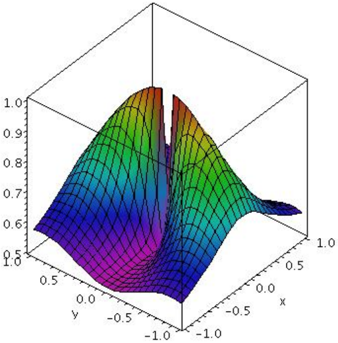


SOLUTION:
Limit along $x = 0$:

$$\lim_{(x,y)\to(0,0)} \left(\frac{\sin(y)}{y}\right)^2 = 1$$

Does the limit exist at $(0,0)$?

$$f(x,y) = \frac{x^2 + \sin^2(y)}{2x^2 + y^2}$$



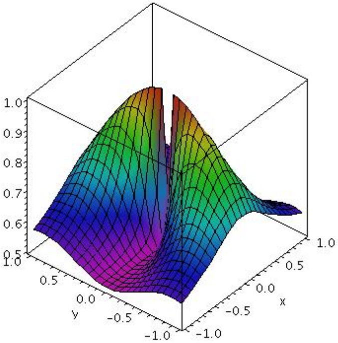
SOLUTION:

Limit along $x = 0$:

$$\lim_{(x,y)\to(0,0)} \left(\frac{\sin(y)}{y}\right)^2 = 1$$

Limit along $y = 0$:

$$\lim_{(x,y)\to(0,0)} \left(\frac{x^2}{2x^2}\right) =$$



Does the limit exist at $(0,0)$?

$$f(x,y) = \frac{x^2 + \sin^2(y)}{2x^2 + y^2}$$

SOLUTION:

Limit along $x = 0$:

$$\lim_{(x,y)\to(0,0)} \left(\frac{\sin(y)}{y}\right)^2 = 1$$

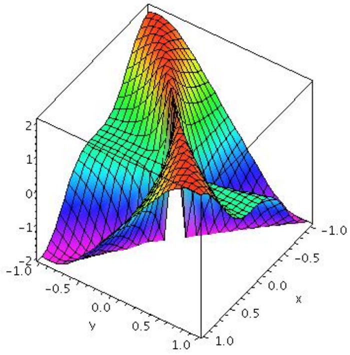
Limit along $y = 0$:

$$\lim_{(x,y)\to(0,0)} \left(\frac{x^2}{2x^2}\right) = \frac{1}{2}$$

The limit DNE at the origin.

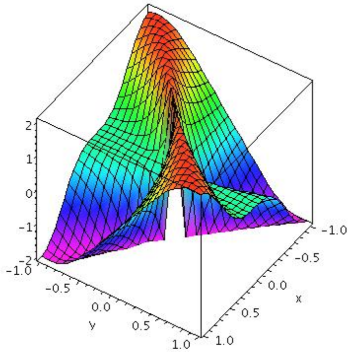
Does the limit exist at $(0,0)$?

$$f(x,y) = \frac{6x^3y}{2x^4 + y^4}$$



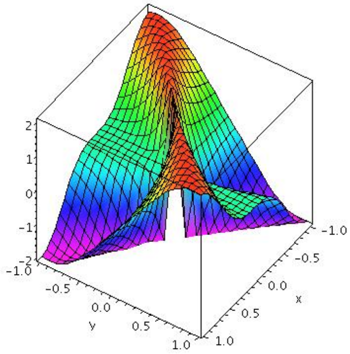
Does the limit exist at $(0,0)$?

$$f(x,y) = \frac{6x^3 y}{2x^4 + y^4}$$



SOLUTION:
Limit along $y = x$:

$$\lim_{(x,y)\to(0,0)} \left(\frac{6x^4}{2x^4 + x^4} \right) =$$

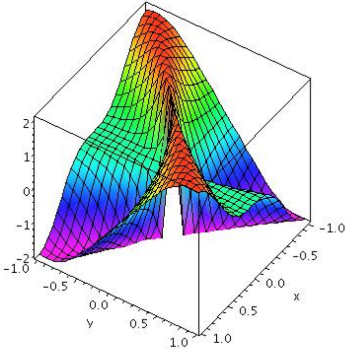


Does the limit exist at $(0,0)$?

$$f(x,y) = \frac{6x^3y}{2x^4 + y^4}$$

SOLUTION:
Limit along $y = x$:

$$\lim_{(x,y)\to(0,0)} \left(\frac{6x^4}{2x^4 + x^4} \right) = 2$$



Does the limit exist at $(0, 0)$?

$$f(x, y) = \frac{6x^3 y}{2x^4 + y^4}$$

SOLUTION:

Limit along $y = x$:

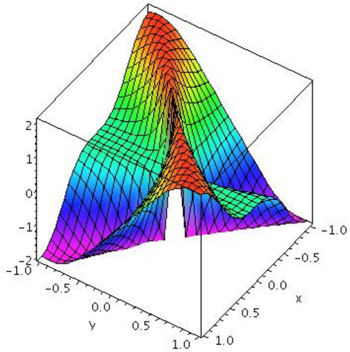
$$\lim_{(x,y)\to(0,0)} \left(\frac{6x^4}{2x^4 + x^4} \right) = 2$$

Limit along $y = -x$:

$$\lim_{(x,y)\to(0,0)} \left(\frac{-6x^4}{2x^4 + x^4} \right) =$$

Does the limit exist at $(0,0)$?

$$f(x,y) = \frac{6x^3y}{2x^4 + y^4}$$



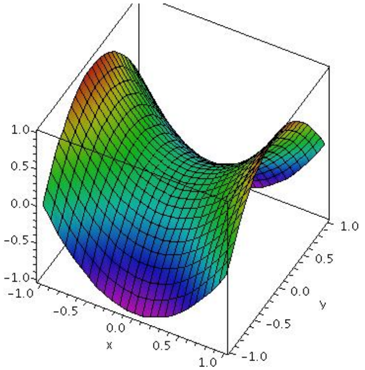
SOLUTION:
Limit along $y = x$:

$$\lim_{(x,y)\to(0,0)} \left(\frac{6x^4}{2x^4 + x^4} \right) = 2$$

Limit along $y = -x$:

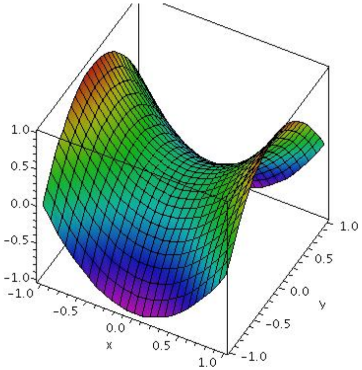
$$\lim_{(x,y)\to(0,0)} \left(\frac{-6x^4}{2x^4 + x^4} \right) = -2$$

The limit DNE at the origin.



Does the limit exist at the origin?

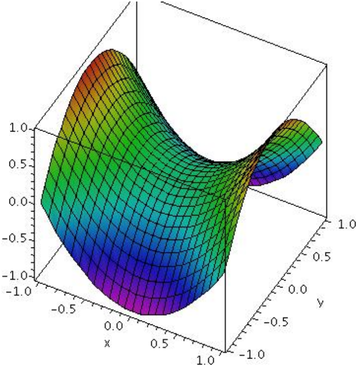
$$f(x, y) = \frac{x^4 - y^4}{x^2 + y^2}$$



Does the limit exist at the origin?

$$f(x, y) = \frac{x^4 - y^4}{x^2 + y^2}$$

SOLUTION: The numerator can be factored

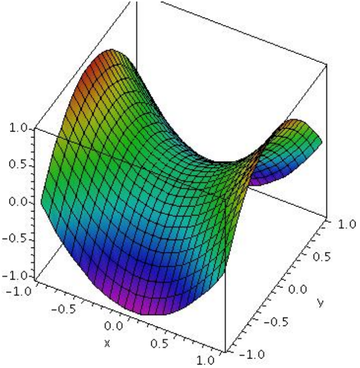


Does the limit exist at the origin?

$$f(x,y) = \frac{x^4 - y^4}{x^2 + y^2}$$

SOLUTION: The numerator can be factored

$$\frac{(x^2+y^2)(x^2-y^2)}{x^2+y^2} = x^2 - y^2$$



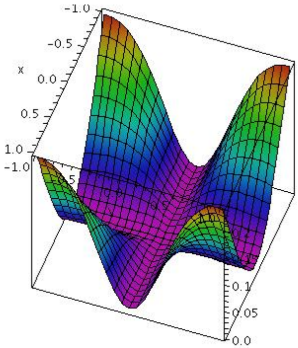
Does the limit exist at the origin?

$$f(x,y) = \frac{x^4 - y^4}{x^2 + y^2}$$

SOLUTION: The numerator can be factored

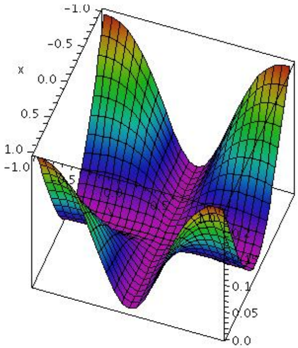
$$\frac{(x^2+y^2)(x^2-y^2)}{x^2+y^2} = x^2 - y^2$$

so the limit is zero.



Does the limit exist at the origin?

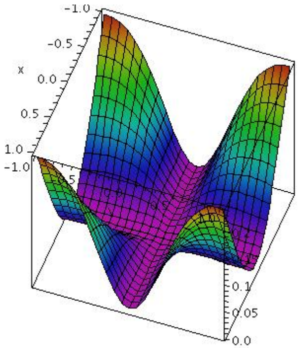
$$f(x,y) = \frac{x^2 \sin^2(y)}{x^2 + 2y^2}$$



Does the limit exist at the origin?

$$f(x, y) = \frac{x^2 \sin^2(y)}{x^2 + 2y^2}$$

SOLUTION:



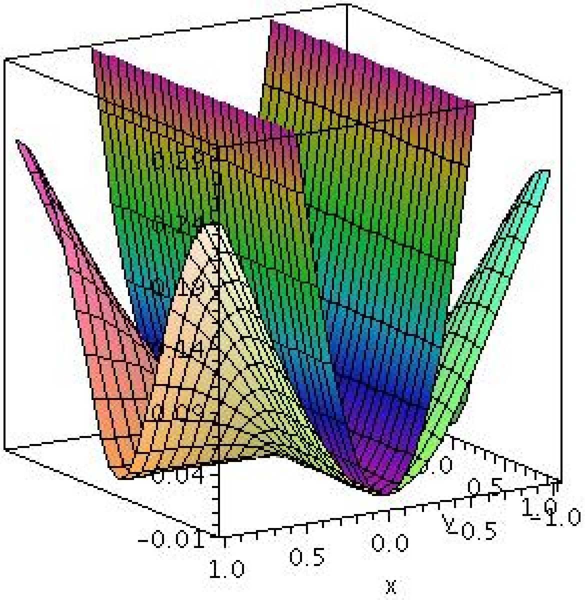
Does the limit exist at the origin?

$$f(x, y) = \frac{x^2 \sin^2(y)}{x^2 + 2y^2}$$

SOLUTION: Squeeze it!

$$0 \leq \frac{x^2 \sin^2(y)}{x^2 + 2y^2} \leq \frac{x^2}{x^2 + 2y^2} \leq x^2$$

By the Squeeze Theorem, the limit is zero.


0.24
0.19
0.14
0.09
0.04
-0.01
1.0
0.5
0.0
-0.5
-1.0
x
y
0.0
0.5
1.0
-1.0
-0.5