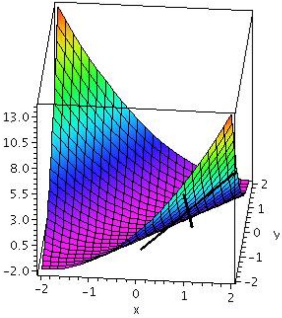
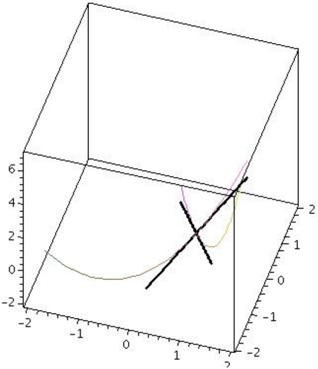
$$z = x^2 - 2xy + y^2 - 2 \text{ at } x = 1, y = -1, z = 2$$





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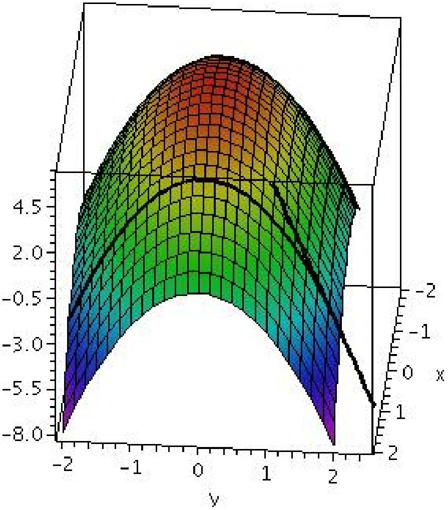
$$x = 1 \quad y = 2 + t \quad z = -4 - 8t$$


4.5
2.0
-0.5
-3.0
-5.5
-8.0
-2
-1
0
1
2
y
-2
-1
0
1
2
x

A Special Function

$$f(x,y) = \begin{cases} \frac{x^3y - xy^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

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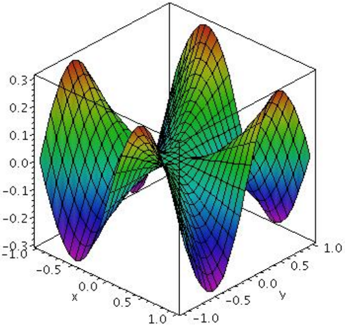

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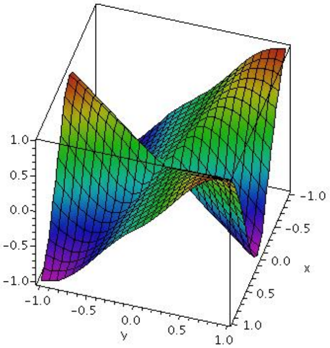

Recall: If $(x, y) \neq (0, 0)$, then

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Does $f_x(0, 0)$ exist? (Use the definition!)

$$f_x(0, 0) = \lim_{h \to 0} \frac{f(0, h) - f(0, 0)}{h} =$$

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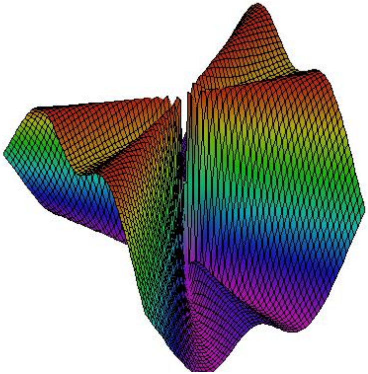

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Therefore,

$$f_{xy}(0, 0) \neq f_{yx}(0, 0)$$

(even though they are equal everywhere else!)