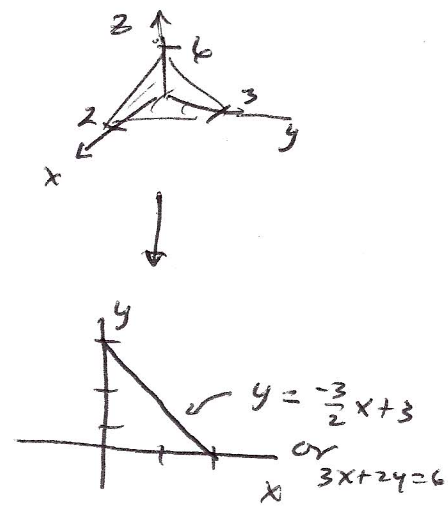
16.6

#37 let $\vec{r}(x,y) = \langle x, y, 6-3x-2y \rangle$



Then $\vec{r}_x = \langle 1, 0, -3 \rangle$

$\vec{r}_y = \langle 0, 1, -2 \rangle$

and $\vec{r}_x \times \vec{r}_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -3 \\ 0 & 1 & -2 \end{vmatrix}$

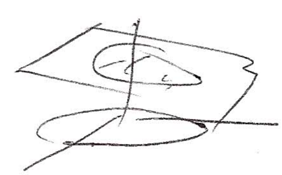
$= \langle 3, 2, 1 \rangle$

$$A(S) = \int_0^3 \int_0^{(6-2y)/3} \sqrt{9+4+1}\, dx\, dy = \int_0^3 \int_0^{2-\frac{2}{3}y} \sqrt{14}\, dx\, dy$$

$= \sqrt{14} \cdot \times$ Area of triangle $= 3\sqrt{14}$

#38 $2x + 5y + z = 10$ inside $x^2+y^2 \leq 9$

$\vec{r} = \langle x, y, 10-2x-5y \rangle$



$\vec{r}_x = \langle 1, 0, -2 \rangle$

$\vec{r}_y = \langle 0, 1, -5 \rangle$

$\vec{r}_x \times \vec{r}_y = \langle 2, 5, 1 \rangle$

$$A(S) = \iint_{x^2+y^2 \leq 9} \sqrt{4+25+1}\, dA = \sqrt{30} \cdot \text{Area of circle radius 3} = 9\sqrt{30}\,\pi$$

39. $z = \frac{2}{3}\left(x^{3/2} + y^{3/2}\right)$ $\quad 0 \le x \le 1$, $0 \le y \le 1$

$$\vec{r}(x,y) = \langle x, y, \tfrac{2}{3}(x^{3/2} + y^{3/2}) \rangle$$

$$\vec{r}_x = \langle 1, 0, \sqrt{x} \rangle, \quad \vec{r}_y = \langle 0, 1, \sqrt{y} \rangle$$

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} i & j & k \\ 1 & 0 & \sqrt{x} \\ 0 & 1 & \sqrt{y} \end{vmatrix} = \langle -\sqrt{x}, -\sqrt{y}, 1 \rangle$$

$$A(S) = \int_0^1 \int_0^1 \sqrt{x + y + 1}\ dx\, dy$$ (Finished w/ set up)

$$= \int_0^1 \tfrac{2}{3}(1 + y + x)^{3/2}\Big|_0^1\ dy$$

$$= \int_0^1 \tfrac{2}{3}(2+y)^{3/2} - \tfrac{2}{3}(1+y)^{3/2}\ dy = \frac{36\sqrt{3} - 32\sqrt{2} + 4}{15} \approx 1.41$$

40 $\vec{r} = \langle 1+v, u-2v, 3-5u+v \rangle$

$$\vec{r}_u = \langle 0, 1, -5 \rangle, \quad \vec{r}_v = \langle 1, -2, 1 \rangle$$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} i & j & k \\ 0 & 1 & -5 \\ 1 & -2 & 1 \end{vmatrix} = \langle -9, -5, -1 \rangle$$

$$A(S) = \int_0^1 \int_0^1 \sqrt{81 + 25 + 1}\ du\, dv = \sqrt{107}$$

#41 The part of $z = xy$ inside $x^2+y^2=1$

$\vec{r} = \langle x, y, xy \rangle$

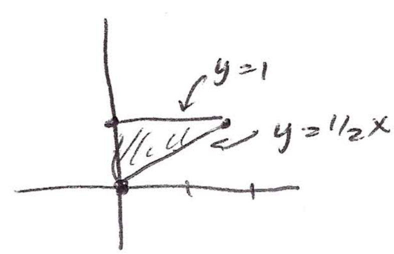
$\vec{r}_x = \langle 1, 0, y \rangle$, $\vec{r}_y = \langle 0, 1, x \rangle$ $\quad \vec{r}_x \times \vec{r}_y = \begin{vmatrix} i & j & k \\ 1 & 0 & y \\ 0 & 1 & x \end{vmatrix}$

$= \langle -y, -x, 1 \rangle$

$\Rightarrow A(S) = \iint_{x^2+y^2 \le 1} \sqrt{1+x^2+y^2}\, dA$ To continue, go to polar coords.

$= \int_0^{2\pi} \int_0^{*} \sqrt{1+r^2}\, r\, dr\, d\theta = 2\pi \cdot \left(\frac{2\sqrt{2}-1}{3}\right)$

#42 The part of $z = 1+3x+2y^2$ above $\longrightarrow$



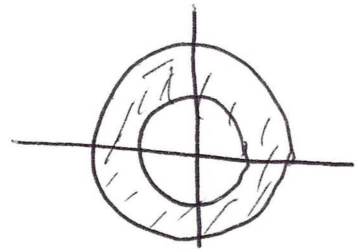
$\vec{r}(x,y) = \langle x, y, 1+3x+2y^2 \rangle$

$\vec{r}_x = \langle 1, 0, 3 \rangle$ $\quad \vec{r}_y = \langle 0, 1, 4y \rangle$

$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 0 & 1 & 4y \end{vmatrix} = \langle -3, -4y, 1 \rangle$

$A(S) = \int_0^1 \int_0^{2y} \sqrt{10+16y^2}\, dx\, dy = \int_0^1 2y\sqrt{10+16y^2}\, dy \approx 4.21$

#43 $\quad z = y^2 - x^2$ between



$$\vec{r} = \langle x, y, y^2 - x^2 \rangle$$

$$\vec{r}_x = \langle 1, 0, -2x \rangle, \quad \vec{r}_y = \langle 0, 1, 2y \rangle$$

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -2x \\ 0 & 1 & 2y \end{vmatrix} = \langle 2x, -2y, 1 \rangle$$

$$A(S) = \iint_{1 \le x^2 + y^2 \le 4} \sqrt{1 + 4x^2 + 4y^2}\, dA$$

To continue, use polar coords.

$$\int_0^{2\pi} \int_1^2 \sqrt{1 + 4r^2}\, r\, dr\, d\theta = 2\pi \left(\frac{17\sqrt{17} - 5\sqrt{5}}{12} \right)$$

#44 The part of $x = y^2 + z^2$ inside $y^2 + z^2 = 9$

In this case, think of (y, z) as the indep. variables. Then

$$\vec{r}(y, z) = \langle y^2 + z^2, y, z \rangle$$

$$\vec{r}_y = \langle 2y, 1, 0 \rangle, \quad \vec{r}_z = \langle 2z, 0, 1 \rangle$$

$$\vec{r}_y \times \vec{r}_z = \langle 1, -2y, -2z \rangle \Rightarrow |\vec{r}_y \times \vec{r}_z| = \sqrt{1 + 4y^2 + 4z^2}$$

So:

$$A(S) = \iint_{y^2 + z^2 \le 9} \sqrt{1 + 4y^2 + 4z^2}\, dA$$

Put in polar form to continue

\# 44, continued...

$$A(S) = \int_0^{2\pi} \int_0^3 \sqrt{1+4r^2}\, r\, dr\, d\theta = 2\pi \left(\frac{37\sqrt{37}-1}{12} \right)$$

\# 45 $\quad y = 4x + z^2, \qquad \left. \begin{array}{l} 0 \le x \le 1 \\ 0 \le z \le 1 \end{array} \right\}$ Treat these as independent.

$$\vec{r}(x,z) = \langle x,\ 4x+z^2,\ z \rangle$$

$$\vec{r}_x = \langle 1, 4, 0 \rangle \qquad \vec{r}_z = \langle 0, 2z, 1 \rangle$$

$$\vec{r}_x \times \vec{r}_z = \begin{vmatrix} i & j & k \\ 1 & 4 & 0 \\ 0 & 2z & 1 \end{vmatrix} = \langle 4, -1, 2z \rangle$$

$$\int_0^1 \int_0^1 \sqrt{17+4z^2}\, dx\, dz$$

(This is as far as we need to go)

\# 46 $\quad \vec{r}(u,v) = \langle u\cos v,\ u\sin(v),\ v \rangle \qquad \begin{array}{l} 0 \le u \le 1 \\ 0 \le v \le \pi \end{array}$

$$\vec{r}_u = \langle \cos v, \sin v, 0 \rangle \qquad \vec{r}_v = \langle -u\sin v, u\cos v, 1 \rangle$$

$$\vec{r}_u \times \vec{r}_v = \langle \sin(v), \cos(v), u \rangle$$

$$\Rightarrow |\vec{r}_u \times \vec{r}_v| = \sqrt{1+u^2} \Rightarrow \int_0^1 \int_0^{\pi} \sqrt{1+u^2}\, du\, dv$$

(ok to stop here)

#47 $x = u^2$, $y = uv$, $z = \frac{1}{2}v^2$ $0 \le u \le 1$

$0 \le v \le 2$

$x_u = 2u$ $y_u = v$ $z_u = 0$

$x_v = 0$ $y_v = u$ $z_v = v$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} i & j & k \\ 2u & v & 0 \\ 0 & u & v \end{vmatrix} = \langle v^2, -2uv, 2u^2 \rangle$$

$$|\vec{r}_u \times \vec{r}_v| = \sqrt{v^4 + 4u^2v^2 + 4u^4} = \sqrt{(v^2 + 2u^2)^2} = v^2 + 2u^2$$

So $A(S) = \int_0^1 \int_0^2 v^2 + 2u^2 \, dv \, du = 4.$