Exam 2 Sample Questions

Be sure to look over your old quizzes and homework as well. For limits, we will provide a graph and contours. No calculators will be allowed for this exam.

1. True or False, and explain:

   (a) There exists a function f with continuous second partial derivatives such that

   $$f_x(x,y) = x + y^2 \qquad f_y = x - y^2$$

   (b) The function f below is continuous at the origin.

   $$f(x,y) = \begin{cases} \frac{2xy}{x^2+2y^2}, & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

   (c) If $\vec{r}(t)$ is a differentiable vector function, then

   $$\frac{d}{dt}|\vec{r}(t)| = |\vec{r}\,'(t)|$$

   (d) If $z = 1 - x^2 - y^2$, then the linearization of z at $(1,1)$ is

   $$L(x,y) = -2x(x-1) - 2y(y-1)$$

   (e) We can always use the formula: $\nabla f(a,b) \cdot \vec{u}$ to compute the directional derivative at (a,b) in the direction of $\vec{u}$.

   (f) Different parameterizations of the same curve result in identical tangent vectors at a given point on the curve.

   (g) If $\vec{u}(t)$ and $\vec{v}(t)$ are differentiable vector functions, then

   $$\frac{d}{dt}\left[\vec{u}(t) \times \vec{v}(t)\right] = \vec{u}\,'(t) \times \vec{v}\,'(t)$$

   (h) If $f_x(a,b)$ and $f_y(a,b)$ both exist, then f is differentiable at (a,b).

   (i) At a given point on a curve $(x(t_0), y(t_0), z_0(t))$, the osculating plane through that point is the plane through $(x(t_0), y(t_0), z(t_0))$ with normal vector is $\vec{B}(t_0)$.

2. Show that, if $|\vec{r}(t)|$ is a constant, then $\vec{r}\,'(t)$ is orthogonal to $\vec{r}(t)$. (HINT: Differentiate $|\vec{r}(t)|^2 = k$)

3. Reparameterize the curve with respect to arc length measuring from $t = 0$ in the direction of increasing t:

   $$\mathbf{r} = 2t\mathbf{i} + (1-3t)\mathbf{j} + (5+4t)\mathbf{k}$$

4. Is it possible for the directional derivative to exist for every unit vector $\vec{u}$ at some point (a,b), but f is still not differentiable there?

   Consider the function $f(x,y) = \sqrt[3]{x^2 y}$. Show that the directional derivative exists at the origin (by letting $\vec{u} = \langle \cos(\theta), \sin(\theta) \rangle$ and using the **definition**), BUT, f is not differentiable at the origin (because if it were, we could use $\nabla f \cdot \vec{u}$ to compute $D_{\vec{u}} f$).

5. If $f(x,y) = \sin(2x+3y)$, then find the linearization of f at $(-3,2)$.

6. The radius of a right circular cone is increasing at a rate of 3.5 inches per second while its height is decreasing at a rate of 4.3 inches per second. At what rate is the volume changing when the radius is 160 inches and the height is 200 inches? ($V = \frac{1}{3}\pi r^2 h$)

7. Find the differential of the function: $v = y\cos(xy)$

8. Find the maximum rate of change of $f(x,y) = x^2 y + \sqrt{y}$ at the point $(2,1)$, and the direction in which it occurs.

9. Find an expression for

$$\frac{d}{dt}\left[\mathbf{u}(t) \cdot (\mathbf{v}(t) \times \mathbf{w}(t)\right]$$

10. Use Lagrange Multipliers to find the maximum and minimum of f subject to the given constraints:

$$f(x,y) = x^2y \qquad x^2 + y^2 = 1$$

11. The curves below intersect at the origin. Find the angle of intersection to the nearest degree:

$$\vec{r}_1(t) = \langle t, t^2, t^9 \rangle \quad \vec{r}_2(t) = \langle \sin(t), \sin(5t), t \rangle$$

12. Find three positive numbers whose sum is 100 and whose product is a maximum.

13. Find the equation of the tangent plane and normal line to the given surface at the specified point:

$$x^2 + 2y^2 - 3z^2 = 3 \qquad (2, -1, 1)$$

14. If $z = x^2 - y^2$, $x = w + 4t$, $y = w^2 - 5t + 4$, $w = r^2 - 5u$, $t = 3r + 5u$, find $\partial z/\partial r$.

15. If $x^2 + y^2 + z^2 = 3xyz$ and we treat z as an implicit function of z, then find $\partial z/\partial x$ and $\partial z/\partial y$.

16. If $\mathbf{a}(t) = -10\mathbf{k}$ and $\mathbf{v}(0) = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{r}(0) = 2\mathbf{i} + 3\mathbf{j}$, find the velocity and position vector functions.

17. Find the equation of the normal line through the level curve $4 = \sqrt{5x - 4y}$ at $(4, 1)$ using a gradient.

18. Find all points at which the direction of fastest change in the function $f(x,y) = x^2 + y^2 - 2x - 4y$ is $\vec{i} + \vec{j}$.

19. Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 6$.

20. Find and classify the critical points:

$$f(x,y) = 4 + x^3 + y^3 - 3xy$$

21. Let $f(x,y) = x - y^2$. Find the gradient of f at $(3, -1)$. We said that this gradient was perpendicular to a level curve of f- Which one? Draw a sketch showing the level curve and the gradient vector, then find the equation of the tangent line to the level curve and the equation of the normal line.

22. Find the equation of the tangent plane to the surface implicitly defined below at the point $(1, 1, 1)$:

$$x^3 + y^3 + z^3 = 9 - 6xyz$$

23. Find parametric equations of the tangent line at the point $(-2, 2, 4)$ to the curve of intersection of the surface $z = 2x^2 - y^2$ and $z = 4$. (Hint: In which direction should the tangent line go?)

24. Find and classify the critical points:

$$f(x,y) = x^3 - 3x + y^4 - 2y^2$$