Note about Exercise 13, Section 10.2

There is some unusual algebra in Exercise 13. Given

$$x = t - \mathrm{e}^t \qquad y = t + \mathrm{e}^{-t}$$

we want to find dy/dx and d^2y/dx^2. In class, we had:

$$\frac{dx}{dt} = 1 - \mathrm{e}^t \quad \frac{dy}{dt} = 1 - \mathrm{e}^{-t} \quad \Rightarrow \frac{dy}{dx} = \frac{1 - \mathrm{e}^{-t}}{1 - \mathrm{e}^t}$$

Now the text says that $dy/dx = -\mathrm{e}^{-t}$- Where does that come from? (It is OK to leave your answer as the fraction by the way):

$$\frac{dy}{dx} = \frac{1 - \frac{1}{\mathrm{e}^t}}{1 - \mathrm{e}^t} = \frac{\frac{\mathrm{e}^t - 1}{\mathrm{e}^t}}{1 - \mathrm{e}^t} = \frac{\mathrm{e}^t - 1}{\mathrm{e}^t} \cdot \frac{1}{1 - \mathrm{e}^t} = \frac{-(1 - \mathrm{e}^t)}{\mathrm{e}^t} \cdot \frac{1}{1 - \mathrm{e}^t} = -\mathrm{e}^{-t}$$

This makes the second derivative much easier to computer!

$$\frac{d^2y}{dx^2} = \frac{d(-\mathrm{e}^{-t})/dt}{dx/dt} = \frac{\mathrm{e}^{-t}}{1 - \mathrm{e}^t}$$

We could still get the answer from what we did in class, but it takes some work to get there. The numerator will be the derivative of $\frac{dy}{dx}$:

$$\frac{\mathrm{e}^{-t}(1 - \mathrm{e}^t) + (1 - \mathrm{e}^{-t})\mathrm{e}^t}{(1 - \mathrm{e}^t)^2} = \frac{\mathrm{e}^{-t} - 2 + \mathrm{e}^t}{(1 - \mathrm{e}^t)^2} = \frac{\frac{1}{\mathrm{e}^t} - 2 + \mathrm{e}^t}{(1 - \mathrm{e}^t)^2} = \frac{\frac{1 - 2\mathrm{e}^t + \mathrm{e}^{2t}}{\mathrm{e}^t}}{1 - 2\mathrm{e}^t + \mathrm{e}^{2t}} = \mathrm{e}^{-t}$$

And the denominator is dx/dt, or $1 - \mathrm{e}^t$.

While the text's answer is very nice, I wouldn't expect you to simplify that far without knowing that it does simplify to something nice; but its good algebra practice to get the answer in the text.