Calc 3

Vector Functions

- Curves $\rightarrow$ Line Integrals $\int_c ds$ $\rightarrow$ Work $\int_c \vec{F}\cdot d\vec{r}$
  - Fund Thm (Conservative)
  - Greens Thm (Closed Path) $\iint \text{curl} F \, dA$
  - Stokes' Thm (Surfaces) $\iint \text{curl} F \, dS$
- Surfaces $\rightarrow$ Surface Integrals $\iint_S dS$ $\rightarrow$ Flux $\iint_S \vec{F}\cdot\vec{n}\, dS$
  - Stokes' Thm (flux of curl) ✓
  - Div Thm (Closed surface) $\iiint \text{div} F \, dV$

Another difference between Stokes' & the Divergence Theorem:

$$\text{Work} = \int_c \vec{F}\cdot d\vec{r} = \int_c \vec{F}\cdot\vec{T}\, ds = \text{measures rotation} = \iint_R Q_x - P_y \, dA = \iint_R \text{curl}(\vec{F})\, dA$$

vs.

$$\int_c \vec{F}\cdot\vec{N}\, ds = \text{measures spread} = \iint_R P_x + Q_y \, dA = \iint_R \text{div}(\vec{F})\, dA$$

(Then Divergence is in 3-d, and $\int_c \vec{F}\cdot\vec{N}\, ds$ becomes a flux integral)

Integrals for Scalar functions: $z = f(x,y)$ or $w = f(x,y,z)$

$\iint_R f(x,y)\, dA$, $\iiint_E f(x,y,z)\, dV$, polar, cylindrical, spherical, general $\Rightarrow$ Jacobian

Key: Change the order of integration, try to look for shortcuts.

Also: $\int_c f(x,y)\, ds$ or $\iint_S f(x,y,z)\, dS$

Derivatives:

Calc 1	Curves	Surfaces
$\frac{dy}{dx} = f'(x)$	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, $\frac{d^2y}{dx} = ?$ $\vec{r}\,'(t)$	$z = f(x,y)$ $D_{\vec{u}} f = \nabla f \cdot \vec{u}$
$dy = f'(x)dx$		$dz = f_x\,dx + f_y\,dy$
$y - f(a) = f'(a)(x-a)$	$\vec{r}(a) + t\vec{r}\,'(a)$ Normal plane: use T as $\vec{n}$ Osculating plane: use $\vec{B}$ as $\vec{n}$	$z - f(a,b) = f_x(a,b)(x-a) + f_y(a,b)(y-b)$
	$F(x,y)=0 \Rightarrow$ find tan line	$F(x,y,z)=0 \Rightarrow$ find tan plane
Chain Rule		z $x \quad y$ etc
Prod Rule	dot & cross	

Continuity & Limits

$\lim_{x \to a} f(x) = f(a)$	$\lim_{(x,y) \to (a,b)} f(x,y)$
Techniques	$\to$ Graph of cont. fcn

Differentiability:

Derivatives: Vector fields

- conservative, div, curl

Maxs & Mins

$y=f(x)$	$z=f(x,y)$ or $w=f(x,y,z)$
EVT Global: CPs, and endpts Local: 1st & 2nd deriv tests	EVT Global: CPs and boundary curve 2nd derivs test $D = f_{xx} f_{yy} - f_{xy}^2$ $D>0 < \begin{matrix} f_{xx}>0 \\ f_{xx}<0 \end{matrix}$ $D<0$ saddle $D=0$ none
	Global; with constraints Lagrange Multipliers max/min f s.t. $g=k$ $\begin{bmatrix} \nabla f = \lambda \nabla g \\ g = k \end{bmatrix}$

~~Geometry & Operations~~:

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Geometry, Operations & Interpretations:

- Relationship of ∇f & level curves of f, & surface f.
- dot & cross
- Magnitude, angle, determinant
- Area of ▱, Vol of
- ortho projections
- distances: pt to plane
- Clairaut's Thm
- div, curl, flux