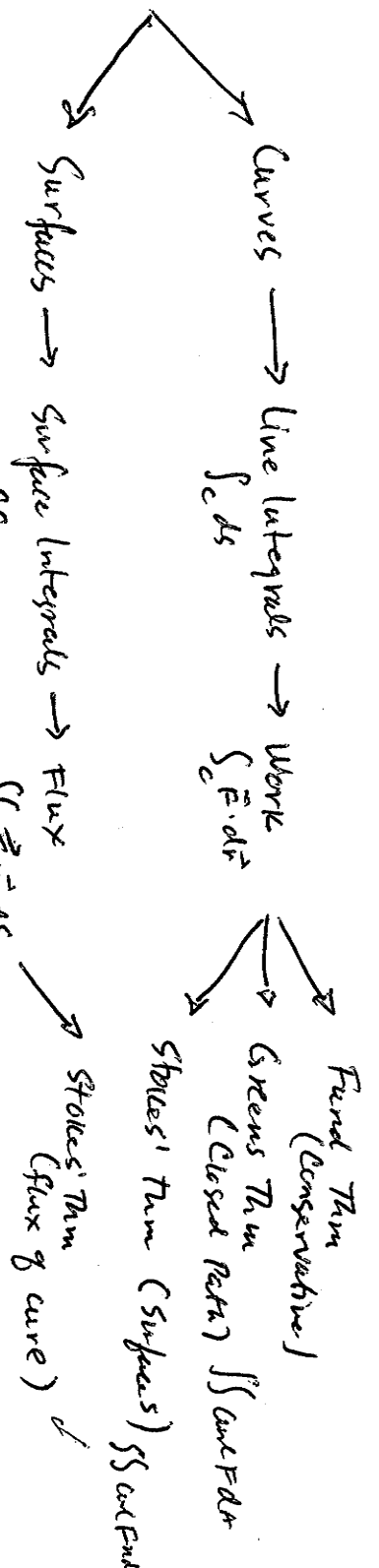


**Calc 3**

Vector Functions



Another difference between Stokes' & the Divergence Theorem:

Work =  $\int_C \vec{F} \cdot d\vec{r} = \int_C \vec{F} \cdot \vec{T} ds = \int_C \text{measures rotation}$  =  $\iint_R \text{curl}(\vec{F}) \cdot d\vec{A}$

vs.  $\int_C \vec{F} \cdot \vec{n} ds = \text{measures spread}$  =  $\iint_R P_x + Q_y \cdot dA = \iint_R \text{div}(\vec{F}) \cdot dA$

(Then Divergence is in 3-d, and  $\int_C \vec{F} \cdot \vec{n} ds$  becomes a flux integral)

Integrals for scalar functions:  $z = f(x,y)$  or  $w = f(x,y,z)$

$\iint_R f(x,y) dA$ ,  $\iiint_E f(x,y,z) dV$ , Polar, cylindrical, spherical, general Jacobian

Key: Change the order of integration, try to look for shortcuts.

Also:  $\int_C f(x,y) ds$  or  $\iint_S f(x,y,z) dS$

Derivatives:

Calc 1

$$\frac{dy}{dx} = f'(x)$$

$$dy = f'(x) dx$$

$$y - f(a) = f'(a)(x - a)$$

Curves

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \quad \frac{d^2y}{dx^2} = ?$$

$$\vec{r}'(t)$$

$$\vec{r}(a) + t\vec{r}'(a)$$

Tangent plane:  
use  $T$  as  $\vec{n}$

Osculating plane:  
use  $\vec{B}$  as  $\vec{n}$

$F(x, y) = 0 \Rightarrow$  find  
tan line

Surfaces

$$z = f(x, y)$$

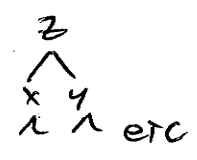
$$\text{D}_u f = \nabla f \cdot \vec{u}$$

$$dz = f_x dx + f_y dy$$

$$z - f(a, b) = f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

$F(x, y, z) = 0 \Rightarrow$  find  
tan plane

Chain Rule



Prod Rule

dot & cross

Continuity & Limits

$$\lim_{x \rightarrow a} f(x) = f(a)$$

$$\lim_{(x, y) \rightarrow (a, b)} f(x, y)$$

Techniques

$\rightarrow$  Graph of cont. fun

Differentiability:

Derivatives: Vector fields

- conservative, div, curl

Max's & Min's

$$y = f(x)$$

EVT

Global: cps,  
and endpoints

Local: 1<sup>st</sup> & 2<sup>nd</sup>  
deriv tests

$$z = f(x, y) \text{ or } w = f(x, y, z)$$

EVT

Global: cps and boundary  
curve

2<sup>nd</sup> derivs test

$$D = f_{xx} f_{yy} - f_{xy}^2$$

$$D > 0 \begin{cases} f_{xx} > 0 \\ f_{yy} < 0 \end{cases}$$

$D < 0$  saddle

$D = 0$  none

Global; with constraints  
Lagrange Multiplier  
max/min  $f$  s.t.  $g = k$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = k \end{cases}$$

## ~~Geometry & Operations:~~

Ge

## Geometry, Operations & Interpretations:

- Relationship  $\nabla f$  & level curves of  $f$ , & surface  $f$ .
- dot & cross
- Magnitude, angle, determinant
- Area of  $\square$ , Vol of  $\square$
- orthog projections
- distance = pt to plane
- Clairaut's Thm
- Div, curl, flux