Exam 3 Review
Math 225

1. Double Integrals (15.1-15.5)

   Use to get volume, average value, geometric interpretation. Fubini's Theorem (15.2), Factoring integrals, Setting up the domain (15.3, Type I and II), Use to get the area, Bounding the integral (max/min using the area), polar coordinate conversions, moments (formulas would be provided).

2. Triple Integrals (15.6-15.9)

   Six setups possible, getting the volume (and center of mass and moments), Cylindrical coordinates, Spherical coordinates (remember the formulas for the volumes!), general coordinate transformations (Jacobian).

3. Vector Fields (16.1): Not too much here. Get a sense of what a vector field is. Vocab: Definition of a vector field, gradient vector field, conservative vector field, potential function.

4. Line Integrals

   Be able to parameterize a curve (especially lines, functions ($y = f(x)$), circles (or ellipses). Be able to estimate the work done by a vector field by traveling along a given path. Graphically estimate if a vector field is conservative.

   Formulas: Arc length formula,

   New notation: What is the meaning of (in terms of how to compute) these. Some of these are alternate notation for the same integral. Which ones?

   $$\int_C f(x,y)\,ds \qquad \int_C f(x,y)\,dx \qquad \int_C f(x,y)\,dy$$
   $$\int_C \vec{F}\cdot\vec{T}\,ds \qquad \int_C P\,dx + Q\,dy \qquad \int_C \vec{F}\cdot d\vec{r}$$

   Alternate forms for computing Work (or the Line Integral): Directly (by the formulas in the notation section), by using the Fundamental Theorem for Line Integrals (be able to construct a potential function), or by **Green's Theorem** (be able to integrate over a region in the plane): $\iint_D Q_x - P_y\,dA$

   Key ideas: Independence of Path, Conservative Vector Field, curl test for conservative vector fields.

5. Alternative ways of computing the area of a region D by taking a line integral (prove using Green's Theorem):

   $$\iint\limits_D 1\,dA = \oint_x dy = -\oint_y dx$$

6. Divergence and Curl:

   Definitions, geometric meaning, irrotational, incompressible,

   Theorems: Curl vs. Conservative Vector Field, div(curl($\vec{F}$),

   If we have the three-dimensional vector field $\vec{F} = \langle P, Q, R \rangle$, then we talked about the divergence, the curl, and the relationship these have with conservative vector fields (the curl is zero). Be able to compute the divergence and curl. We could also use the divergence to test to see if a curl is valid (div(curl($\vec{F}$))=0).

7. 16.6/16.7: Surfaces we consider: $z = f(x, y)$, planes, spheres, cylinders and/or boxes.

   Formulas for area and volume: The area of a parallelogram defined by vectors $\vec{a}$ and $\vec{b}$ in three dimensions is $|\vec{a} \times \vec{b}|$ (The cross product is only defined for 3-d). The volume of the parallelepiped defined by $\vec{a}, \vec{b}, \vec{c}$ is given by $\vec{a} \cdot (\vec{b} \times \vec{c})$.

   The surface area of surface S (using a domain D in u, v) can be written in terms of $\vec{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle$:

   $$\iint_S dS = \iint_D |\vec{r}_u \times \vec{r}_v| \, dA$$

   The integral $\displaystyle\iint_S g(x, y, z) \, dS$ means that we are looking at the effect of the function g across the surface S (so to integrate it, substitute $z = f(x, y)$ in g). Compare this to $\int_C f(x, y) \, ds$.

   Finally, we have the flux (think about volume of fluid through the surface):

   $$\iint_D \vec{F} \cdot (\vec{r}_u \times \vec{r}_v) \, dA = \iint_S \vec{F} \cdot \vec{n} \, dS = \iint_S \vec{F} \cdot d\vec{S}$$

   The unit normal

   $$\vec{n} = \frac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|} \qquad dS = |\vec{r}_u \times \vec{r}_v| \, dA$$

   Compare these to the earlier notions of the unit tangent and arc length:

   $$\vec{T} = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} \qquad ds = |\vec{r}\,'(t)| \, dt$$

8. The important thing here is to understand what the notation means. We have these expressions for the flux because we don't want to have to specify how the surface is defined.

   If we did specify it, we could get specific formulas like on pg. 10, but it is easier to use the general terms. To help you with notation, here is an old quiz that focused on it.

Old Quiz

1. If the curve C is parameterized by $\langle t^2 - t, 2t + 4\rangle$, then compute:
   (a) $ds =$
   (b) $d\vec{r} =$
   (c) Set up the arc length integral for $0 \le t \le 1$.
2. If the surface S is parameterized by $\langle x, y, 3x^2 - xy + 5\rangle$ then compute:
   (a) $\vec{r}_x \times \vec{r}_y =$
   (b) $d\vec{S} =$ $\qquad\qquad$ $dS =$
   (c) Set up the integral for the surface area over the rectangle $0 \le x \le 3, -1 \le y \le 2$
   (d) The surface normal, $\vec{n} =$
   (e) If the vector field $\vec{F} = \langle x, y, z^2\rangle$, set up the integral: $\iint_S \vec{F} \cdot d\vec{S}$
3. Is $\displaystyle\int_C \vec{F} \cdot d\vec{r} = \int_C f(x, y)\, ds$? Explain.
4. Is $\displaystyle\iint_S g(x, y, z)\, dS = \iint_S \vec{F} \cdot d\vec{S}$? Explain.
5. Is $\int_C \vec{F} \cdot d\vec{r} = \int_C P\, dx + Q\, dy$?
6. Is $\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n}\, dS$?
7. Given surface S over domain D, is $\iint_S \vec{F} \cdot dS = \iint_D \vec{F} \cdot (\vec{r}_x \times \vec{r}_y)\, dA$?
8. Set up the integral (DO NOT EVALUATE) representing the flux of $\vec{F}$ across the surface S, if the orientation is upward, and
   $$\vec{F} = \langle y, x, z^2\rangle \qquad z = 4 - x^2 - y^2 \qquad 0 \le x \le 1, 0 \le y \le 1$$
9. Set up the integral for the surface area of $4x - 2y + 2z = 4$ above the unit circle in the plane.
10. Set up the integral $\iint_S y\, dS$, if the surface is given by the part of the cone $x^2 + y^2 = z^2$ that lies between the planes $z = 1$ and $z = 3$.