

Concept Review

Vector Functions, $\mathbf{r}(t)$, (Chap 13)

Note: No questions about curvature

- Definitions: Continuous, “smooth”, $\mathbf{T}(t)$, $\mathbf{N}(t)$, $\mathbf{B}(t)$, velocity, speed, acceleration, $s(t)$ (the arc length function), and its derivative, ds/dt .
- Formulas: $\frac{d\mathbf{r}}{dt}$, the equation of the tangent line (in two and three dimensions), the three “product rules” on p. 826, Example 4 p. 827.
- Computations: Derivative, integral, find velocity and position given acceleration, construct the arc length function, construct the normal plane and osculating plane.
- Parameterizations: Be able to construct parameterizations for lines (or line segments), circles, functions of the form $y = f(x)$, intersections of planes or more general surfaces (like 36-38, p. 823)

“Derivatives” and $z = f(x, y)$ or $w = f(x, y, z)$ (Ch 14)

- Concepts to understand: The level curve (contours) or level surface, The difference (esp. in notation) between an implicitly defined surface and an explicitly defined surface. A saddle point (versus a local max or local min).

The relationships between the gradient, the partial derivatives, and the directional derivative. Be able to show that the direction in which the directional derivative is maximum is in the direction of the gradient.

Estimate the partial derivatives (e.g., 70 on p. 890). Estimate the directional derivative (e.g., 18 on p. 920).

Understand and illustrate the following ideas: “The gradient is perpendicular to the level curves of f ”. “The gradient is perpendicular to the surface of F ”.

Why is a limit more difficult to compute now than it was in Calc I? What does it mean (intuitively) for a function to be differentiable at a point (a, b) ? How do we go about showing that f is differentiable at (a, b) ?

- (Formal) Definitions: Continuity. The partial derivatives at a point (a, b) . The directional derivative of f at (a, b) in the direction of the unit vector $\langle u_1, u_2 \rangle$.

NOTE: For limits, a graph of the surface and its level curves would be provided. For continuity, only use the definition on p. 874 and NOT the one on p. 876.

- Know these Theorems: Clairaut’s Theorem. The Second Derivatives Test, The Extreme Value Theorem.

- Computations: Use the Squeeze Theorem for limits. Be able to compute the directional derivative using the definition and using the “shortcut formula” with the gradient. Be able to find and classify critical points based on the Second Derivatives Test.
- Find the global (or absolute) maxima and minima if f is a continuous function on a closed and bounded domain.
- Find the maximum and minimum for a given function given a constrained domain (like $g(x, y) = c$) using the Method of Lagrange Multipliers.