Exam 1 Sample Questions

Be sure to look over your old quizzes and homework as well. For more review questions, see the Chapter 12 Review, pg. 812-813.

1. Eliminate the parameter to find a Cartesian equation for the curve: $x = e^t$, $y = 2e^t$.

   Find the arc length of the curve from $t = 1$ to $t = 2$ of the original parametric form, then for the (x, y) form.

2. Find dy/dx and d^2y/dx^2, if $x = t^3 - 12t$ and $y = t^2 - 1$

3. A plane is given as $2x + 3y + 4z = 12$. Sketch it in the first octant by drawing the lines of intersection between it and the coordinate planes.

   Give the equations of the lines of intersection that you have drawn (in parametric form).

4. Convert the polar equation to Cartesian: $r = \tan(\theta)\sec(\theta)$

5. Convert the equation from Cartesian to polar of the form $r = f(\theta)$: $xy = 4$

6. Find the area of the surface obtained by rotating the curve about the $x-$axis:

$$x = 3t - t^3 \qquad y = 3t^2 \qquad 0 \le t \le 1$$

7. Find the slope of the tangent line to the given polar curve at the point specified by θ:

$$r = 2 - \sin(\theta) \qquad \theta = \frac{\pi}{3}$$

8. Find the distance between the point $(1, 2, 3)$ and the plane $x + y + z = 1$.

9. Find the distance between the planes $x + y + z = 1$ and $3x + 3y + 3z - 5 = 0$.

10. Find the angle between the vectors $\langle 1, 2, 3\rangle$ and $\langle 1, 0, 1\rangle$. Find the projection of the first vector onto the second.

11. If a child pulls a sled through the snow on a level path with a force of 50 N exerted at an angle of 38° with the horizontal, find the work in pulling the sled 10 meters.

12. Graphical problems: For example p. 627, and Ex. 49-50, p. 648, and p. 777.

13. Do the lines below intersect? If so, find the point of intersection. If not, find the distance between them.

$$x = 3 + t, y = 2 - t, z = 1 \qquad x = 2 + s, y = 1 + 2s, z = 2 - s$$

14. Find an equation for the surface obtained by rotating the parabola $y = x^2$ about the $y-$axis. (Hint: In 3-d, if you fix a $y-$value, what shape should you have in the $xz-$plane?)

15. Write the parametric equations for each situation:

    (a) Go from $(1, 2)$ to $(-3, 2)$ as time runs from 0 to 1.

(b) Go around the unit circle twice in clockwise fashion starting at $(-1, 0)$ as $0 \le t \le 1$.

(c) Follow the curve $y = 3x^2 + 5x + 2$ as $-\infty < t < \infty$.

(d) Follow the curve $r = 1 + 2\cos(\theta)$ (in the Cartesian coordinate system) as $0 \le \theta \le 2\pi$.

16. Find the error (if there is one): Let $\mathbf{u}$ be a vector such that $|\mathbf{u}| = \sqrt{2}$. Choose a vector $\mathbf{v} \neq \mathbf{u}$ such that $\mathbf{u} \cdot \mathbf{v} = 2$. Now we have:

$$\begin{aligned}
2(\mathbf{u} \cdot \mathbf{u}) &= \mathbf{u} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{v} \\
2(\mathbf{u} \cdot \mathbf{u}) - 2(\mathbf{u} \cdot \mathbf{v}) &= \mathbf{u} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{v} - 2(\mathbf{u} \cdot \mathbf{v}) \\
2(\mathbf{u} \cdot \mathbf{u}) - 2(\mathbf{u} \cdot \mathbf{v}) &= \mathbf{u} \cdot \mathbf{u} - \mathbf{u} \cdot \mathbf{v} \\
2\mathbf{u} \cdot (\mathbf{u} - \mathbf{v}) &= \mathbf{u} \cdot (\mathbf{u} - \mathbf{v}) \\
2\mathbf{u} &= \mathbf{u} \\
2 &= 1
\end{aligned}$$

17. Show that $r = 2\cos(\theta)$ is a circle by converting it into Cartesian coordinates and writing it in standard form.

18. Find the scalar and vector projections of $\mathbf{a}$ onto $\mathbf{b}$, if

- $\mathbf{a} = \langle -2, 3, -6 \rangle \qquad \mathbf{b} = \langle 5, -1, 4 \rangle$
- $\mathbf{a} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{b} = \mathbf{j} - \mathbf{k}$

19. Sketch a few traces of the surface $y^2 + z^2 = 1 + x^2$ and describe the resulting surface (in words and/or as a sketch in three-dimensions).

20. Let a line L and a plane P be defined as:

$$L: \quad x = \frac{y+2}{3} = -z \qquad P_1: \quad x + y + z = 1$$

(a) Find the point of intersection between the plane and the line.

(b) Given the point $Q(1, 2, 1)$, find the plane P_2 that contains the line L and the point Q.

(c) Find the distance between Q and the plane P_1. What is the distance between planes P_1, P_2?

21. Find the error in the following: Let $\mathbf{u}$ be a vector such that $|\mathbf{u}| = 1$. Choose a vector $\mathbf{v}$ such that $\mathbf{u} \cdot \mathbf{v} = 3$ and $|\mathbf{v}| = \sqrt{5}$. Now we have:

$$\begin{aligned}
|\mathbf{u} - \mathbf{v}|^2 &= (\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) \\
&= \mathbf{u} \cdot \mathbf{u} - 2(\mathbf{u} \cdot \mathbf{v}) + \mathbf{v} \cdot \mathbf{v} \\
&= 0
\end{aligned}$$

Therefore, $\mathbf{u} - \mathbf{v} = \vec{0}$, and so $\mathbf{u} = \mathbf{v}$.