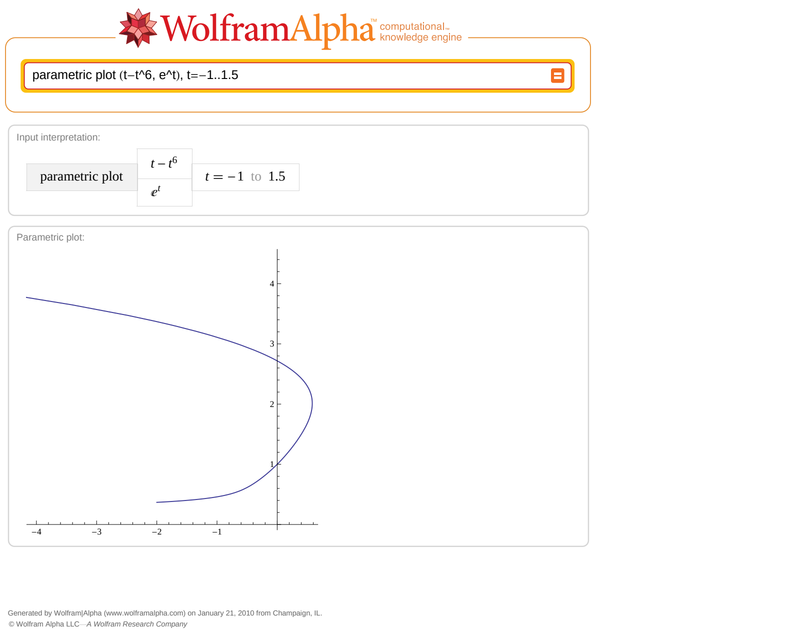
Selected Solutions, Section 10.2

21. For the graph, use your calculator or Wolfram Alpha:



From this, we can estimate the position at about $(0.6, 2)$. To find the exact coordinates, we find the value of t that will give us a vertical tangent line- That will be where $dx/dt = 0$:

$$1 - 6t^5 = 0 \quad \Rightarrow \quad t = 6^{-1/5}$$

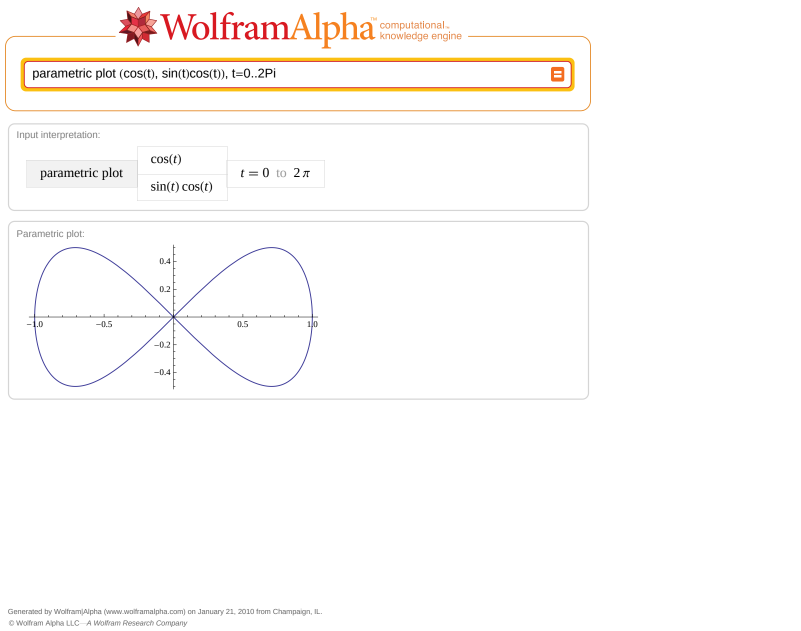
Inserting this into the formulas for x, y give the coordinates:

$$(6^{-1/5} - 6^{-6/5}, e^{6^{-1/5}}) \approx (0.582, 2.01)$$

25. Given the plot,

We have two tangent lines. To find the times, set x and y equal to zero. We'll only look for solutions in the interval $(0, 2\pi)$.

$$x = 0 \quad \Rightarrow \quad \cos(t) = 0 \quad \Rightarrow \quad t = \frac{\pi}{2}, \frac{3\pi}{2}$$



Will y also be zero at these times? Yes, since $y = \sin(t)\cos(t)$- In fact, y will also be zero at times 0 and 2π, but we don't need these points (in these cases, $x = \pm 1$).

Now, find dy/dx for $t = \pi/2$ and $3\pi/2$ by first computing dx/dt and dy/dt:

$$\frac{dx}{dt} = -\sin(t) \qquad \frac{dy}{dt} = \cos^2(t) - \sin^2(t)$$

At $t = \pi/2$:

$$\frac{dy}{dx} = \frac{0^2 - 1^2}{-1} = 1$$

Similarly, at $t = 3\pi/2$,

$$\frac{dy}{dx} = \frac{0^2 - (-1)^2}{-(-1)} = -1$$

So the tangent lines are $y = x$ and $y = -x$, which make sense with our graph.

36(a). The area is twice the area inside R that is above the x axis. The top half of the loop is described by:

$$x = t^2, \quad y = t = t^3 - 3t \qquad -\sqrt{3} \le t \le 0$$

Now perform substitution

$$A = 2\int_0^3 y\,dx = 2\int_0^{-\sqrt{3}} (t^3 - 3t)(2t\,dt) = 2\int_0^{-\sqrt{3}} 2t^4 - 6t^2\,dt = \cdots = \frac{24\sqrt{3}}{5}$$

36(b). We do something similar, using the formula for disks:

$$V = \pi\int_0^3 y^2\,dx = \pi\int_0^{-\sqrt{3}} (t^3 - 2t)^2(2t\,dt) = 2\pi\int_0^{-\sqrt{3}} (t^6 - 6t^4 + 9t^2)\,dt = \cdots = \frac{27\pi}{4}$$

37. The important part is to be able to set up the integral. In this case, $x = t - t^2$, and $y = \frac{4}{3}t^{3/2}$ for $1 \le t \le 2$. Now,

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \sqrt{(1 - 2t)^2 + 4t}\,dt = \sqrt{1 + 4t^2}\,dt$$

Therefore, the arc length is given by

$$\int_1^2 ds = \int_1^2 \sqrt{1 + 4t^2}\,dt$$

If your calculator is not set up to approximate integrals, use Wolfram Alpha, and type:

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integrate (sqrt(1+4t^2)), t=1..2
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And you'll get an answer of approximately 3.16. (NOTE: I will not ask you to numerically approximate an integral on an exam or quiz, but it is useful to know for homework).

39. This uses the same ideas as 37.

41. In this case, we hope that the arc length integral is integrable, and it is- You should get

$$L = \int_0^1 \sqrt{36t^2 + 36t^4}\,dt = \int_0^1 6t\sqrt{1 + t^2}\,dt$$

Then use $u = 1 + t^2$, $du = 2t\,dt$, etc.