Pre-Lab: Taylor Series

1. A *power series based at* $x = a$ is a function of the form:

$$P(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$$

where c_n is a constant, for all n. What does it mean to take an "infinite" sum?

2. What is the radius of convergence for a power series? What is the "Ratio Test"?

3. Can you differentiate/integrate a power series? What is the result after doing this to $P(x)$ defined in Problem 1?

4. What is a Taylor series? How is it different than a Maclaurin series?

5. What is the meaning of "n^{th} Taylor polynomial"? What is the meaning of $R_n(x)$ (used in Stewart's Calc text)?

6. How does using a Taylor series allow us to integrate functions we otherwise could not?

7. Do *all* functions have a power series representation? How can you tell if it does?

8. It is shown in Calculus that, if $|x| < 1$, then

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \tag{1}$$

Show graphically that the partial sums approximate $1/(1-x)$ by plotting some. That is, define:

$$f_0(x) = \frac{1}{1-x}, \quad f_3(x) = 1 + x + x^2 + x^3,$$

$$f_6(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6, \text{ etc.}$$

and plot f_0 along with functions of increasing degree. Careful in defining the domain and range windows!

9. This refers to the previous problem: Integrate both sides of Equation 1 and repeat the graphs.