Lab 3: Clairaut's Theorem[1]

A famous theorem from Calculus is the theorem of Clairaut- That mixed second partial derivatives are equal. If you haven't gotten to that section yet in the Calculus course, look it up in the index. Because most functions we work with are "nice", it is easy to think that Clairaut's Theorem applies to *every* function. One purpose of the lab is to get you looking at functions of more than one variable in Maple- feel free to do any appropriate graphing to help you in this lab. A lab checklist has been included on the back of this sheet so that you can see what the grading criteria will be.

Recall from your text[2]

Theorem 0.1 (Clairaut)**.** *Suppose f is defined on a disk D that contains the point (a,b). If the functions f_{xy} and f_{yx} are both continuous on D, then*

$$f_{xy}(a,b) = f_{yx}(a,b).$$

Consider the function

$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases}$$

- Is f continuous? What are f's domain and range? Can looking at a contour plot help you guess if a function is continuous? Try comparing f with something that is continuous, like g_1 below, and with something that is not continuous, like g_2 below:

$$g_1(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2} & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases} \qquad g_2(x,y) = \begin{cases} \frac{x^2}{x^2+y^2} & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases}$$

- Show that $f_y(a,0) = a$ for all a and $f_x(0,b) = -b$ for all b.
- Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.
- Discuss why Clairaut's Theorem does not apply here.
- Include in your write-up one substantive extension topic.

[1]Calculus On Manifolds by M. Spivak, adaptation of exercise 24, page 29

[2]Stewart, 4th ed., page 902