Lab 1: Maple Questions

This is the second part of Lab 1

1. When we measure things, like heights of individuals, Intelligence Quotient (IQ) scores, and so on, the numbers we get tend to distribute themselves in a way very similar to a "bell curve". That is, most of the numbers you collect will probably be very similar to the mean of the data, with some numbers getting larger or smaller.

   The "bell curve", or *normal distribution* is modeled by the function:

   $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)} = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

   (a) Using Maple, plot $f(x)$ for different values of μ, holding σ fixed. What is the effect of changing μ?

   (b) Using Maple, plot $f(x)$ for different values of σ, holding μ fixed. What is the effect of changing σ

   (c) Here we explain why the fraction $\frac{1}{\sigma\sqrt{2\pi}}$ appears in the front of $f(x)$. Have Maple compute the integral:

   $$\int_{-\infty}^{\infty} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx$$

   Hint: It might be easiest to let $a = \mu$, $b = \sigma$, and use the command `assume(b>0);` to get a numerical answer.

   Note that $\displaystyle\int_{-\infty}^{t} f(x)\, dx$ is a number ranging from 0 to 1, so we can interpret this as a *probability.*

   Definition: The probability that a measurement X is between numbers a and b is defined as:

   $$P(a \leq X \leq b) = \int_a^b f(x)\, dx$$

   For the following questions, we are measuring IQ scores. The mean μ is given by 100, and the "standard deviation", σ is 15. For these questions, we will need to get an *estimate* of the integral.

   (d) What percentage of the population has an IQ score between 85 and 115?

   (e) What percentage of the population has an IQ score greater than 140?

2. Let

   $$x(t) = \int_1^t \frac{\cos(u)}{u}\, du, \quad y(t) = \int_1^t \frac{\sin(u)}{u}\, du$$

   First, plot the curve for t ranging between 1 and 50. Second, find the length of the arc made by $(x(t), y(t))$ for $t = 1$ to $t = 10$.