The Bicycle Race [1]

Carol is a local bicycle racing star and today she is in the race of her life. Moving at a constant velocity k meters per second, she passes a refreshment station. At that instant ($t = 0$ seconds), her support car starts from the refreshment station to accelerate after her, beginning from a dead stop. Suppose the distance traveled by Carol in t seconds is given by the expression kt and distance traveled by the support car is given by the function $\frac{1}{3}(10t^2 - t^3)$, where distance is measured in meters. This latter function is carefully calculated by her crew so that at the instant the car catches up to the racer, they will match speeds. A crew member will hand Carol a cold drink and the car will immediately fall behind.

1. How fast is Carol traveling?

2. How long does it take the support car to catch her?

3. Suppose that Carol is riding at a constant velocity k, which may be different than the value found in part (a). Find an expression for the times when the car and the bike meet which gives these times as a function of her velocity k. How many times would the car and bike meet if Carol were going faster than the velocity found in part (a)? Slower?

4. Consider a pair of axes with time measured horizontally and distance vertically. Draw graphs that depict the distance traveled by Carol and by the car plotted on the same axes for the original problem (parts (a) and (b)) and for the questions of part (c). You should have three graphs: one for the bike's velocity found in part (a), one for a faster bike, and one for a slower bike. If Carol had been going any faster or slower than the velocity you found in part (a), passing the drink would not have been so easy. Why? Justify your answer.

5. Prove that if a cubic polynomial $P(x)$ has a double root at $x = a$, then $P'(a) = 0$. How does this relate to your answer for part (a) and to your graphs in part (d)?

[1] from Student Research Projects in Calculus, Cohen, M, et al.