Lab 2: The Spirograph

September 20, 2006

In this lab, we use the Spirograph (a child's toy from Hasbro) to investigate parametric curves.

We'll simplify the problem to a small circle rotating about a larger, fixed, circle. There is a pen attached to the smaller circle (denoted by an asterisk in Figure 1). Define R to be the radius of the large circle, r is the radius of the small circle, and $r + h$ is the "radius" to the pen. Furthermore, we have two angles- Let t be the angle for the large circle and θ is the angle for the smaller circle.

Here are the lab questions. Your write up should be a discussion that incorporates the answers- Don't just list them! See the **sample write** (on the class website) up for an example. You might also see the grading sheet to get an idea of how the lab will be graded (also online).

1. Come up with parametric equations, $x(t)$ and $y(t)$, for the path of the pen in the Spirograph. There are multiple ways of defining t- For this lab, let us assume that t is the angle coming from the large, fixed, circle. Here are some notes that might help you get started:

   (a) Given a circle of radius r, the (x, y) coordinates of a point on its boundary are $(r\cos(\theta), r\sin(\theta))$, where θ is the central angle (measured from the positive $x-$axis).

   (b) The arc length s from a circle of radius r, and measured from a central angle θ: $s = r\theta$

   (c) Try first writing the coordinates of the rotating circle. Define t as the angle for the fixed circle, and θ be the angle for the small circle. What are the coordinates of the pen?

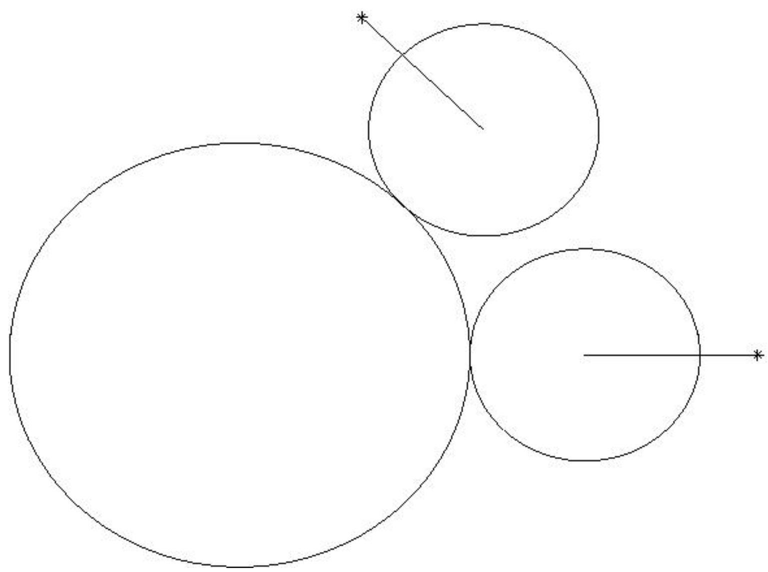

Figure 1: The set up for the spirograph.

2. Come up with some nice patterns! You should try different end values of t.

3. Do all the curves end up being periodic (in the sense that the curve is closed)? Can you describe when you will get closed curves? (Experiment with different values of R and r).

4. Give the general formula for the arc length of the pen in the spirograph. Give a numerical value for the arc length of a closed curve (from your previous example).

5. You can bring in some of the mathematical names of these curves. You might do a little internet research to see if there is anything interesting about them.

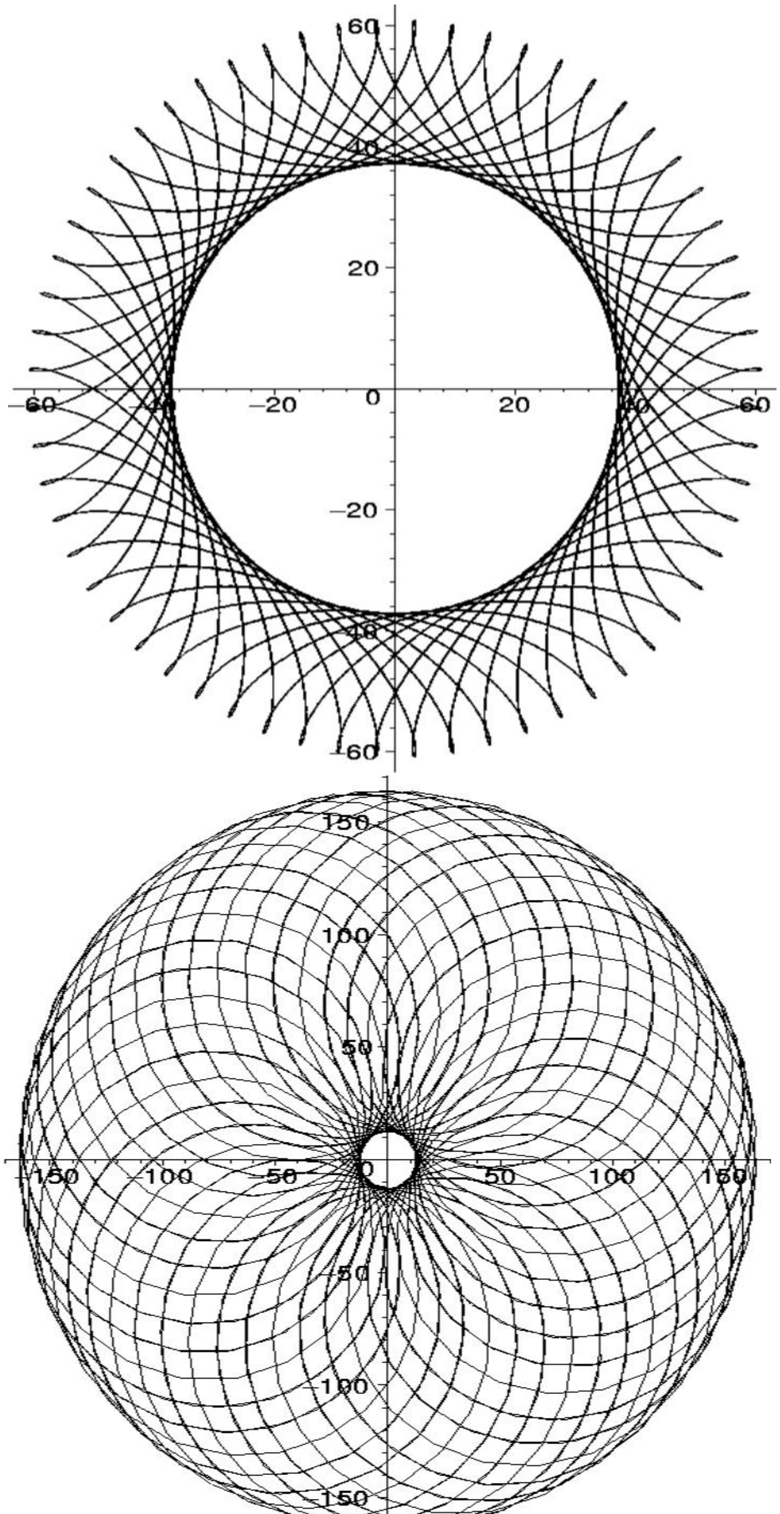

Figure 2: Two examples.