LAB 3: THE SPIROGRAPH

In this lab, we use the Spirograph (a child's toy from Hasbro) to investigate parametric curves.

Suppose you begin with a circle of radius R centered at the origin, C. Take another circle of radius r, label it $\hat{C}$, as shown in Figure 1. A felt pen is connected to $\hat{C}$, and it rotates about C. The pen leaves a trace- a sample trace is given in Figure .

Here are the lab questions. Your write up should be a discussion that incorporates the answers- Don't just list them! See the grading rubric for more information.

(1) Come up with parametric equations, $x(t)$ and $y(t)$, for the path of the pen in the Spirograph.
(2) Come up with some nice patterns! You should try different end values of t.
(3) The parametric curve $(x(t), y(t))$ is said to be closed (or the functions are periodic) if there is a T so that $(x(t), y(t)) = (x(T), y(T))$.

    Can you make a proposition about what might need to be true about the relative circumferences of the circles? Give a specific example.
(4) Give the general formula for the arc length of the pen in the spirograph. Give a numerical value for the arc length of a closed curve (from your previous example).
(5) In the "Conclusions" section, you can bring in some of the mathematical names of these curves. You might do a little internet research to see if there is anything interesting about them. Is there such a thing as an extension to three dimensions, for example?

Some Maple examples:

- A sample parametric curve in two dimensions. The "numpoints" option forces Maple to interpolate through more data points, resulting in a smoother plot (the default is 50).

```
x:=(t*cos(t)-sin(t))/(1+t^2);
y:=cos(2*t);
plot([x,y,t=0..50],numpoints=1000);
```

- You can also use

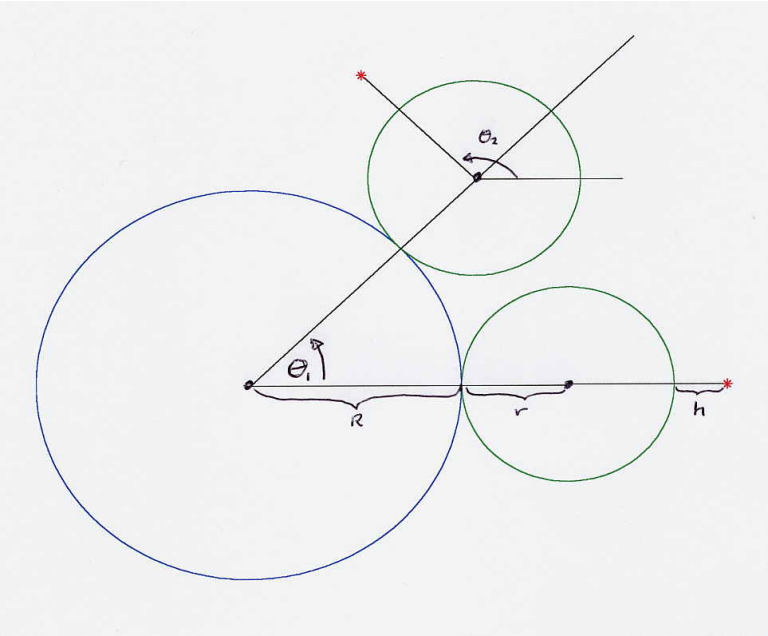

FIGURE 1. The set up for the spirograph.

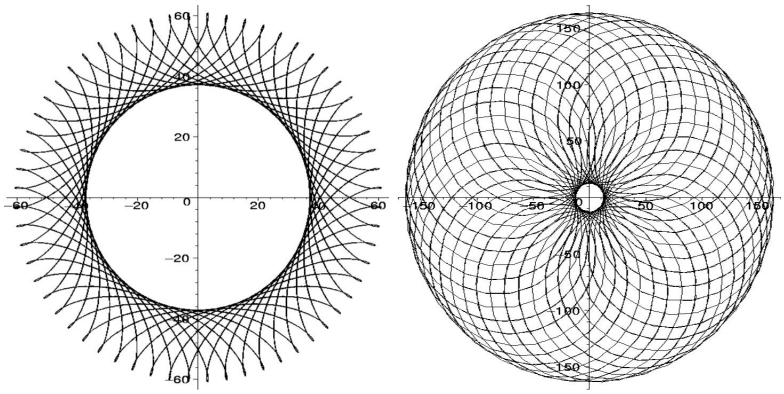

FIGURE 2. Two examples.