Lab 2: Series Expansions

Your Name Here

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1 Introduction to the Lab

In the pre-lab, we discussed the Taylor expansion for a function at a point, $x = a$ as:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$$

We can view this as a decomposition of $f(x)$ into pieces, or building blocks, which are 1, x, x^2, x^3, etc, and the Taylor expansion tells us how to compute the constants that put the blocks together.

Are there other sets of polynomials that can be used to approximate functions? In this lab, we look at a popular set- The Chebyshev polynomials.

Suppose that we define

$$T_0(x) = 1,\ T_1(x) = x$$

and the remaining polynomials $T_k(x)$ are defined recursively:

$$T_k(x) = 2xT_{k-1}(x) - T_{k-2}(x)$$

so that, for example, $T_2(x) = 2xT_1(x) - T_0(x) = 2x^2 - 1$ and $T_3(x) = 2xT_2(x) - T_1(x) = 4x^3 - 3x$

Definition: The set of polynomials, $T_0, T_1, \ldots$ are called *Chebyshev polynomials.*

We won't prove this, but if we have a function $f(x)$ defined on the interval $x \in [-1, 1]$, then we can write:

$$f(x) = a_0T_0(x) + a_1T_1(x) + a_2T_2(x) + \ldots$$

where the coefficients a_k are computed by:

$$a_0 = \frac{1}{\pi}\int_{-1}^{1} \frac{f(x)T_0(x)}{\sqrt{1-x^2}}\,dx,\ a_k = \frac{2}{\pi}\int_{-1}^{1} \frac{f(x)T_k(x)}{\sqrt{1-x^2}}\,dx \text{ for } k > 1$$

When you compute this in Maple, use -1.0 and 1.0 as the bounds- Maple will return numerical approximations to the integrals, which are fine.

2 The Lab Assignment

Compare and contrast the Maclaurin series and Chebyshev polynomials as methods for approximating a function f on the interval $-1 \le x \le 1$. As a starting point for your discussions, try to approximate $f(x) = e^x$ with either Maclaurin series up to degree k or Chebyshev polynomials up to degree k, and see which function is closer to e^x for $x \in [-1, 1]$ for each k. Try some different functions (remember that the approximation is only for $-1 \le x \le 1$) and see if the same thing happens- can you give some possible reasons for your findings?

You might find the following useful: In general, if $p(x)$ is an approximation to $f(x)$, then the error is given by $|p(x) - f(x)|$, and this can be plotted in Maple as: `plot(abs(p(x)-f(x)),x=-1..1);` You may estimate the maximum error off of the graph.

Schedule and Grading Criteria

We will have our full session next week to look at the Maple code we need, and to start typing in LaTeX. You might go ahead and work up a LaTeX document before we meet so that you just need to put in the Maple results.

The lab will not be until until the end of the week after next, before class on October 8th/9th (depending on your lab day).

There will be a specific grading rubric posted next week, but some things we will be looking for: Insert figures into your document, use a “do loop”, and the use of a sequence (or list).