Math 235: Calculus Lab

Prof. Doug Hundley

Whitman College

Weeks 7-8

Groups For Clairaut Lab (1:00PM, Wed)

- Row 1 (Front Row)
  - Group 1: Devon Yee, Connor Hargus
  - Group 2: Isaac Berez, Brock Wade
- Row 2:
  - Group 3: Kaitlin Puryear, Matt Buswell
  - Group 4: Matt Ryan, Joanna Gonda
  - Group 5: Colin McCarthy, Shenjun Wang
- Row 3:
  - Group 6: Jaime Paredes-Torres, Clayton Over
  - Group 7: Emily Dotts, Hallie Barker
- Row 4:
  - Group 8: Braden Hussey, Dalton Cooper
  - Group 9: Riley Jordan, Keith Eubanks

Groups For Clairaut Lab (2:30PM, Wed)

- Row 4 (Back Row)
  - Group 1: Emmanuel James, Alix Eisenbrey
  - Group 2: Ricardo Vivanco, Daniel Kim
- Row 3:
  - Group 3: Kanupria Sanu, Tom Motzer
  - Group 4: Tim Grote, Yarden Blausapp
- Row 2:
  - Group 5: Dylan Zukin, Will Mullins
  - Group 6: Zach Turner, Godwin Wang
- Row 1 (Front Row)
  - Group 7: Catie Chun, Moustafa El Badry Shaker

Overview of Lab

Clairaut's theorem (Section 14.3 of online book):

Suppose f is defined on a disk D that contains the point (a, b). If the functions f_{xy} and f_{yx} are both continuous on D, then

$$f_{xy}(a, b) = f_{yx}(a, b).$$

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$$f(x, y) = 3x^2 y + x \sin(y)$$

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$$f(x, y) = 3x^2y + x\sin(y)$$

$$f_x(x, y) = 6xy + \sin(y) \qquad f_y(x, y) = 3x^2 + x\cos(y)$$

$$f_{xx} = 6y \qquad f_{xy} = 6x + \cos(y) \qquad f_{yx} = 6x + \cos(y) \qquad f_{yy} = -x\sin(y)$$

$f_{xx} = 6y \qquad f_{xy} = 6x+\cos(y) \qquad f_{yx} = 6x+\cos(y) \qquad f_{yy} = -x\sin(y)$

In Maple:

```
F:=3*x^2*y+x*sin(y)
```

First derivatives:

```
Fx:=diff(F,x);    Fy:=diff(F,y);
```

Second derivatives:

```
Fxx:=diff(F,x$2);   Fyy:=diff(F,y$2);
```

Mixed second derivatives:

```
Fxy:=diff(F,x,y);   Fyx:=diff(F,y,x);
```

Example

Compute the partial derivative of F with respect to x at the point $(3, 1)$ by using the *definition* of the derivative (in Maple).

$$F_x(3,1) = \lim_{h \to 0} \frac{F(3+h,1) - F(3,1)}{h}$$

$$= \lim_{h \to 0} \frac{(3(3+h)^2 + (3+h)\sin(1)) - (27 + 3\sin(1))}{h}$$

```
F:=(x,y)->3*x^2*y+x*sin(y);
F1:=(F(3+h,1)-F(3,1))/h;
F2:=limit(F1,h=0);
```

Similarly, we can define F_{xy}:

$$F_{xy}(3,1) = \lim_{h \to 0} \frac{F_x(3, 1+h) - F_x(3,1)}{h}$$

where $F_x = 6xy + \sin(y)$.

To get several graphs on one figure, you can put `includegraphics` for each graph. For example

```
\begin{figure}[h]
\centering
\includegraphics[width=2.0in]{Lab02Fig01}\qquad
\includegraphics[width=2.0in]{Lab02Fig01}
\caption{This is a caption for the figure.}
\label{LabelForGraph01}
\end{figure}
```

See the result in the PDF version.

For the bibliography, here's an example- Put it at the end where you want the bib to appear.

```
\begin{thebibliography}{9}

\bibitem{Erdos01} P. Erd\H os, \emph{A selection
of problems and results in combinatorics}, Recent
trends in combinatorics (Matrahaza, 1995), Cambridge
Univ. Press, Cambridge, 2001, pp. 1--6.

\bibitem{ConcreteMath}
R.L. Graham, D.E. Knuth, and O. Patashnik,
\emph{Concrete mathematics}, Addison-Wesley,
Reading, MA, 1989.

\bibitem{Knuth92} D.E. Knuth, \emph{Two notes on notation},
Math. Monthly \textbf{99} (1992), 403--422.

\end{thebibliography}
```

Now in the text, include something like:

```
This is obvious \cite{Erdos01}.
```

Which results in: This is obvious [1]. If you see a question mark for the citation, run LaTeX again.

As you go through the lab:

- Think about what it means (graphically) for a function to be continuous (taking a limit in the plane).
- Compute partial derivatives in Maple and by using the definition.
- Understand why a certain function fails to satisfy the hypotheses of Clairaut's Theorem.
- Write up your thoughts. Be sure to include references and figures! Use the template to get you started.

- P. Erdős, *A selection of problems and results in combinatorics*, Recent trends in combinatorics (Matrahaza, 1995), Cambridge Univ. Press, Cambridge, 2001, pp. 1–6.
- R.L. Graham, D.E. Knuth, and O. Patashnik, *Concrete mathematics*, Addison-Wesley, Reading, MA, 1989.
- D.E. Knuth, *Two notes on notation*, Amer. Math. Monthly **99** (1992), 403–422.