THE HYPERBOLIC FUNCTIONS

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1. Definitions

In this lab, we explore the functions known as the hyperbolic functions.
Definition: The hyperbolic sine function is defined as:

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

Similarly, the hyperbolic cosine is defined as:

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

(1) Use Maple to plot $\sinh(x)$, $\frac{1}{2}e^x$, $-\frac{1}{2}e^{-x}$.
(2) Use Maple to plot $\cosh(x)$, $\frac{1}{2}e^x$, $\frac{1}{2}e^{-x}$.
(3) Use Maple's `animate` command to determine how the parameter a changes the graph of $\sinh(ax)$ and $\cosh(ax)$. (Look up the use of `animate` using Maple's help)
(4) Look up the definition of *even* and *odd* functions. What do these mean graphically? Show (graphically) that the sine and hyperbolic sine are odd. Show (graphically) that both the cosine and hyperbolic cosine are even.
(5) Using the definitions from trigonometry, how should the hyperbolic tangent, hyperbolic cotangent, hyperbolic secant, and hyperbolic cosecant be defined? Write them in terms of $\sinh(x)$ and $\cosh(x)$, then as exponential functions.
(6) Show that $\cosh^2(x) - \sinh^2(x) = 1$ (This is a bit different than the original trig functions).
(7) Are the hyperbolic sine, cosine and tangent functions 1-1? If not, restrict the domain so that each is 1-1.
(8) Since the hyperbolic trig functions are written in terms of exponentials, we can write the inverse hyperbolic trig functions in terms of logarithms. Compute the inverse hyperbolic sine cosine and tangent in terms of logarithms. Hint: In general, how do we find an inverse?

2. Calculus with the Hyperbolic Functions

(1) Calculate the derivatives of $\sinh(x)$, $\cosh(x)$, and $\tanh(x)$. Write your final answers in terms of hyperbolic functions (rather than exponentials).
(2) Have Maple calculate the derivatives of the inverse hyperbolic sine, cosine and tangent.
(3) Plot $\sinh(x)$ together with its derivative.
(4) Plot $\cosh(x)$ together with its inverse.

3. Applying the Hyperbolic Functions

(1) A flexible cable strung between two telephone poles always hangs in the shape of a catenary curve, defined by:

$$y(x) = c + a\cosh(x/a)$$

where c, a are constants and $a > 0$. Choose a value of c and determine how the value of a changes the graph. (Use Maple's `animate` command).

(2) In physics, it can be shown that when a cable is hung between two poles, it takes the shape of a curve y that satisfies the differential equation:

$$y'' = \frac{pg}{T}\sqrt{1 + (y')^2}$$

where y' is $\frac{dy}{dx}$, p, g, T are constants. Show that

$$y = \frac{T}{pg}\cosh\left(\frac{pgx}{T}\right)$$

is a solution to the differential equation (That is, actually substitute the formula given for y into the differential equation, and show that it is true.).