Lab 6 Questions:

1. Let $y'' + 6y = \delta(t-4)$, $y(0) = 0, y'(0) = 0$

   (a) Solve and plot the solution.

   (b) If we think of this solution as the position of a spring at time t, then putting $\delta(t-4)$ on the right hand side of the equation means that we're giving the spring a "hit" at time 4 with a total impulse of 1 (impulse being the integral of the Dirac function).

   If we let the motion continue, can you hit the spring at some future time to cancel the motion out (so the solution goes to zero again, and stays zero)? Explain, show the new solution, and give another plot.

2. Let $y'' + \frac{1}{2}y' + 2y = 0$, $y(0) = 0, y'(0) = 1$, so that we've added some damping. Algebraically, denote the solution by $F(t)$ (so we don't get too many y's).

   (a) Compare the solution to the original ODE with the solution to:

   $$y'' + \frac{1}{2}y' + 2y = \delta(t-2), \quad y(0) = 0, y'(0) = 1$$

   Compare these two solution with the solution to:

   $$y'' + \frac{1}{2}y' + 2y = \delta(t-2) + \delta(t-4), \quad y(0) = 0, y'(0) = 1$$

   What is the effect on the solution by adding on something of the form $a \cdot \delta(t-k)$? Try to write your answer in terms of F.

   (b) The original solution represents a damped sine wave. Is it possible to time and dose the "hits" so that the spring always returns to the same height? First, explain how you approach this problem, then try to do it for at least the first two periods.

3. Let $y'' + 6y = \delta(t)$, $y(0) = 0, y'(0) = 0$.

   (a) First, use Maple to solve it directly using `dsolve`

   (b) Now solve it "manually" like in the example sheets by first having Maple compute the Laplace transform, solve for $Y(s)$, then invert the transform. Is this the same solution? If not, which solution is correct?

   (c) Compare the correct solution to the ODE with the solution to:

   $$y'' + 6y = 0, \qquad y(0) = 0, y'(0) = 1$$

   What do you see (algebraically)?