# Chapter 3

# Lab 2: Solving some Exercises

For the second lab, we'll look at a very popular application of linear systems- Finding a model function given some data.

First, suppose that we have 4 data points given as ordered pairs:

$$\begin{array}{c|cccc} x & -1 & 0 & 1 & 2 \\ y & 1 & 2 & -1 & 1 \end{array}$$

And we want to find the cubic polynomial that intersects those four points. The general form of a cubic is given as:

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x_3$$

Now, to determine the values of the constants $a_0, a_1, a_2, a_3$, we insert the data into the equation, getting one equation per data point:

$$\begin{array}{rl} a_0 + a_1(-1) + a_2(-1)^2 + a_3(-1)^3 & = 1 \\ a_0 + a_1(0) + a_2(0)^2 + a_3(0)^3 & = 2 \\ a_0 + a_1(1) + a_2(1)^2 + a_3(1)^3 & = -1 \\ a_0 + a_1(2) + a_2(2)^2 + a_3(2)^3 & = 1 \end{array}$$

Rewriting the system as an augmented matrix:

$$\left[\begin{array}{rrrr|r} 1 & -1 & 1 & -1 & 1 \\ 1 & 0 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 8 & 1 \end{array}\right]$$

You might notice the columns of the matrix- If we let $\mathbf{x} = (-1, 0, 1, 2)$, then the columns are (in Matlab notation): `x.^0, x.^1, x.^2, x.^3`. That always happens when we use full polynomial as a model (in this case, a cubic).

```
x=[-1;0;1;2]
A=[x.^0 x x.^2 x.^3]
b=[1;2;-1;1]
C=rref([A b])
% The coefficients are the last column of C
```

```
t=linspace(-1,2);
yy=C(1,5)-C(2,5)*t-C(3,5).*t.^2+C(4,5).*t.^3;
plot(x,b,'*',t,yy)
```

The homework for Matlab is to:

1. Solve 1.2.33, and plot the results in a similar fashion to the previous example.
2. Solve 1.4.38, 40 (Remember to use `C1S4`!)
3. Solve 1.4.41, 42.

For your Matlab script, here's a template to get you started. Remember to publish your script, then upload the PDF file to CLEo. The due date will be set in class.

```
%% Homework Set 2
% Solve and plot 1.2.33, then solve 1.4.38, 40, 41, 42.

%% 1.2.33


%% 1.4.38


% and so on....
```