Group Work, 1.7

1. Determine by inspection if the given column vectors are linearly independent:

   (a) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ -1 & 0 & 3 & 2 & 8 \\ 0 & 2 & 3 & 1 & 0 \\ -1 & 5 & 2 & -1 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 3 \\ 3 & 0 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 3 \\ -1 & 1 \end{bmatrix}$

2. Let the row reduced echelon form of A be given by the matrix below:

   $$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

   (a) Do the columns of A span $\mathbb{R}^3$?

   (b) Do the columns of A span $\mathbb{R}^2$?

   (c) Are the columns of A linearly independent?

   (d) Which set of columns are linearly independent?

3. Let the row reduced echelon form of A be given by the matrix below:

   $$\begin{bmatrix} 1 & 0 & 0 & 2 & 1 & 3 \\ 0 & 1 & 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & 5 & 1 & 1 \end{bmatrix}$$

   (a) Do the columns of A span $\mathbb{R}^3$?

   (b) Are the columns of A linearly independent?

   (c) Can we write the fifth column as a linear combination of the others? Be explicit.

4. If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ form a linearly independent set of vectors, is it true that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ must also be linearly independent? Hint: Consider the equation

   $$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 + 0\mathbf{v}_4 = \vec{0}$$

5. Suppose A is $m \times n$ with the property that for all $\mathbf{b} \in \mathbb{R}^m$, the equation $A\mathbf{x} = \mathbf{b}$ has at most one solution. Must the columns of A be linearly independent or dependent?