Overview: Linear Algebra

- The final examination is comprehensive but it focuses on the main concepts and topics studied during the semester. Since many topics build on earlier material, a majority of the questions will involve ideas from Chapters 4, 5, and 6. For percentages, about 40% of the exam will be the "new" material, and 60% spread across the older material.
- The theme of the first two chapters is to look at systems of equations from many different points of view (system of equations, vector equation, matrix equation, and linear transformation). It might help to review the various questions that involve existence and uniqueness of solutions. The third chapter was all about the determinant (and Cramer's Rule).
- There may be a few definitions on the exam. The most important definitions include:

  The product $A\mathbf{x}$, the product AB, linear transformation, linear independence and dependence (in a general vector space), spanning set, basis, subspace, dimension, rank, orthogonal basis, eigenvector (eigenspace), eigenvalue, least-squares solution, similar matrices. Most recently, we defined an inner product in general, and the inner product on $C[a,b]$.
- We had the four fundamental subspaces of a matrix. Related is the kernel of a linear transformation. Given a matrix, find a basis for all except the null(A^T) using the RREF. Given a matrix, find a basis for all 4 subspaces using the SVD.

  Discuss $A\mathbf{x} = \mathbf{b}$ using the 4 subspaces. For example, if there is a unique solution, in which subspaces must $\mathbf{x}, \mathbf{b}$ be? How about if there is an infinite number of solutions? How about if there is no solution? Continuing this, is it possible for the least squares solution to have an infinite number of solutions?
- We looked at Markov chains, and they served to motivate the concept of the eigenvalue and eigenvector.
- Matrix factorizations: QR factorization, diagonalization (as PDP^{-1} and PCP^{-1}, and the more general SVD, $U\Sigma V^T$).
- A number of questions will require that you give reasons for your answers. These reasons will often involve a reference to a theorem. Theorems that have descriptive names attached to them are usually good candidates for a question. Examples:
  - The Basis Theorem (Theorem 12, 4.5),
  - The Rank Theorem (Theorem 14, 4.6),
  - The Diagonalization Theorem (Theorem 5, 5.3) and its complex variant (Theorem 9, 5.5),
  - The Orthogonal Decomposition Theorem (Theorem 8, 6.3),
  - The Best Approximation Theorem (Theorem 9, 6.3),
  - Projection into a Subspace (Theorem 10, 6.3)
  - The Gram-Schmidt Process (Theorem 11, 6.4)
  - Of particular importance is the Invertible Matrix Theorem (Theorem 8, 2.3; Theorem 4, 3.2; Ending of Section 4.6, Section 5.2 before Theorem 3 and 7.4 just before Example 7).
- You won't need a calculator for the exam (no calculators allowed). Most of the computational questions will come in your take-home portion of the exam.
- Be sure to review your old exams and the review sheets for them.