Some Additions to Review Set 2

1. What are the two ways we defined matrix multiplication, AB? (Hint: One way was the rwo-column rule).

2. Let A, B be given below. Form the matrix product AB, if defined:

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 1 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$$

3. Given the matrix A, B below:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 0 \\ 3 & 2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

   (a) Compute only the $(2,3)$ entry of AB:

   (b) Compute only the $(3,2)$ entry of AB^T:

   (c) Compute $B - 3I_3$:

   (d) Compute C_{23} for matrix A (that's the (2,3) cofactor).

4. If A is the 2×3 matrix below, find a matrix C so that $AC = I$, but note that C is not the inverse of A. To simplify your computations, I've given you one form for C that you might use.

$$A = \begin{bmatrix} -1 & 2 & -1 \\ 6 & -9 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ 0 & 0 \end{bmatrix}$$

5. Suppose A is $n \times n$ with the property that $A\mathbf{x} = \vec{0}$ has only the trivial solution. Without using the invertible matrix theorem, explain directly why the equation $A\mathbf{x} = \mathbf{b}$ must have a solution for every $\mathbf{b}$.

6. Explain why the columns of A^2 span $\mathbb{R}^n$ whenever the columns of A are linearly independent. (Hint: You might think about whether or not A^2 must be invertible).

7. Suppose subspace H is the span of the two vectors below in set $\mathcal{B}$:

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \right\} = \{\mathbf{v}_1, \mathbf{v}_2\}$$

   (a) Does $\mathcal{B}$ span $\mathbb{R}^3$? Why or why not?

   (b) Find $[\mathbf{v}_1]_{\mathcal{B}}$

   (c) If $\mathbf{c} = (3, 3, 0)$, find $[\mathbf{c}]_{\mathcal{B}}$