Review, Exam 1: Math 240 (Linear Algebra)

We looked at Sections 1.1-1.5, 1.7-1.9. In this part of the course, we began by looking at systems of linear equations, and learned how to solve them via row reduction to RREF. Once we got some experience solving and interpreting the RREF, we then learned vocabulary and ideas that will allow us to generalize our results in a more abstract context (for later).

Key Definitions

Be able to define the following terms: linear combination, $A\mathbf{x}$ (as a linear combination), the span, linear independence, linear transformation, standard matrix of a linear transformation.

We reviewed some terms that deal with *functions* generally: domain, codomain, range, image, preimage, linear, one-to-one and onto.

Key Theorems

Theorem 2 (Existence and Uniqueness), Theorem 3 (Matrix equation), Theorem 4 (Cols of A and span), Theorem 5 (Props of matrix-vector product), Theorems 7-9 (Props of lin dep sets), Theorem 10 (How to construct the standard matrix), Theorem 11-12 (we called these "The onto theorem" and "The one-to-one theorem").

Key Skills

- Know the three row operations, and the algorithm we use to get a matrix to RREF.
- Determine when a system is consistent. Know that the general solution can be written as $\mathbf{x}_h + \mathbf{x}_p$ (and what those mean). Write the general solution in parametric vector form.
- Determine values of parameters that make a system consistent, or make the solution unique. Describe existence or uniqueness of solutions in terms of pivot positions. Determine when a homogeneous system has a nontrivial solution.
- Determine when a vector is in a subset spanned by specified vectors. Exhibit a vector as a linear combination of specified vectors. Determine if a vector is in the range of a linear transformation.
- Write a parametric equation of a line through $\mathbf{p}$ and $\mathbf{q}$. Write the parametric equation of a line through $\mathbf{p}$ in a direction parallel to $\mathbf{a}$. Write the equation of a plane in parametric vector form.
- Determine whether the columns of an $m \times n$ matrix span $\mathbb{R}^m$. Determine whether the columns are linearly independent.
- Determine if a given function is a *linear* transformation.
- Use linearity of matrix multiplication to compute $A(\mathbf{u} + \mathbf{v})$ or $A(c\mathbf{u})$.
- Find the standard matrix of a linear transformation.
- Determine whether a linear transformation is one-to-one and whether the transformation is onto.