Content for Exam 2, Linear Algebra

Exam 2 will cover material from 2.1-2.3 (Inverse matrices), 3.1-3.3 (determinants), 4.1-4.6 (vector spaces).

Sections 2.1-2.3

1. Skills

   - Compute the inverse of a 2×2 matrix A directly (Theorem 4)
   - Compute the inverse of an $n \times n$ matrix using row reduction.
   - Compute the elementary matrix for a given row operation.
   - Solve a matrix equation using inverses.

2. Know the Invertible Matrix Theorem (Theorem 8)

   That is, you do not need to be able to list all of the parts, but given a prompt, be able to finish the statement so that it is equivalent to A being invertible. For example, "What is true about the columns of A?" Answer might be: Columns are linearly independent, Columns are pivot columns, Columns span $\mathbb{R}^n$ (any of these).

3. Theorems: Understand Theorem 5, 6, 7 (be able to compute using them). You do not need to know Theorem 9 (p 131).

Sections 3.1-3.2, some 3.3

1. Skills

   - Be able to compute determinants using a cofactor expansion along any row or column.
   - Compute a determinant for upper or lower triangular matrix.
   - Be able to compute a determinant by first performing row reduction.
   - Use Cramer's Rule to solve a system

2. Properties of the determinant. For the following, assume E, A, B are square matrices. For the last item, assume A is invertible.

   (a) Elementary matrices:
   - E corresponding to a row swap: $\det(E) = -1$
   - E corresponding to multiplying a row by k: $\det(E) = k$
   - E corresponding to $kr_j + r_i \rightarrow r_i$: $\det(E) = 1$

   (b) General properties:
   - A is invertible only if $\det(A) \neq 0$.
   - If A is $n \times n$, then $\det(kA) = k^n \det(A)$.
   - $\det(AB) = \det(A)\det(B)$
   - $\det(A^T) = \det(A)$
   - $\det(A^{-1}) = 1/\det(A)$

3. Theorems

   - Theorems 1, 2, 3 and 9 are used in computation. You do not need to memorize these, you can just use them.
   - You should know Theorems 4, 5, 6, 7. These are summarized in the properties, and Theorem 7 is Cramer's Rule.
   - Theorem 8 (A formula for A^{-1} involving the "adjoint" will not be on the exam, neither will the adjoint. Cofactors (book notation: C_{ij}) will be used in computing determinants.
   - Theorem 10 will not be on the exam (the area of an image).

Vector Spaces, (4.1-4.6)

1. You don't need to memorize the 10 axioms on page 217.
2. Be familiar with some template vector spaces: $\mathbb{R}^n$, $\mathbb{P}_n$, $\mathbb{P}$, $C[a,b]$, $M_{m\times n}$
3. Know these definitions: A subspace, a linearly independent set, a basis, the coordinates of $\mathbf{x}$ (with respect to a given basis), the dimension of a subspace, an isomorphism, the rank of a matrix. The four fundamental subspaces associated with a matrix A (be able to define each one), the kernel of a transformation, the change of coordinates matrix.
4. Theorems for computation: 1, 4, 5, 6, 7, 9, 10, 11, 13.

   These are theorems that you should know for computational purposes (you might think of them as "basic facts").
5. Theorems to know: 2 (Null space is a subspace), 3 (Col space is a subspace), 8 (Isormorphism for Isomorphic Spaces), 12 (The basis theorem), 14 (The rank theorem).
6. Skills:
   - Prove that a given set is or is not a subspace.
   - Given a matrix A, be able to compute a basis for the column space, the null space and the row space (not the null space of A^T).
   - Find the kernel of a given transformation and describe the range of the transformation.
   - Understand how row operations effect the the 4 fundamental subspaces (for example, the subspaces for a matrix A versus its RREF, B).
     - Row operations do not effect the relationship among the columns of A, but they do effect the column spaces (the column spaces of A, B may not be the same).
     - Row operations do effect the relationship among the rows of A, but the row spaces of A, B are the same.
     - Row operations do not effect the set of solutions to $A\mathbf{x} = \mathbf{0}$, so the null spaces of A, B are the same.
   - Find the coordinates of a vector given a basis (both in $\mathbb{R}^n$ using the change of coordinates matrix, and for vector spaces that are not $\mathbb{R}^n$, like $\mathbb{P}_n$.
   - Be able to compute the dimension of a vector space.
   - Be able to compute the rank of a matrix. Use that to compute the dimensions of the four fundamental subspaces.
   - Understand what it means to say that two vectors spaces are **isomorphic**.
   - Theorem: Any finite dimensional vector space V (with dimension n) is isomorphic to $\mathbb{R}^n$ using the coordinate mapping as the isomorphism.