Take Home Portion: Final Exam for Math 240, Spring 2016

Instructions: You may work with one partner and you may use your textbook, class notes, and the material from our class website (write the name of the person you work with at the top the script file). You should not use any other people or materials.

What to turn in: See the video on the class website. Basically, "publish" your Matlab work using the "Publish" button in the Matlab editor, then when you go to print, save it as a PDF document. If you don't see that as an option, go ahead and print as usual and slide the hard copy under my office door. If you have an electronic (PDF) copy, upload it to your CLEo box.

Due date: Tuesday, May 17, 11PM.
NOTE: I can answer questions about the material we have looked at in the past, but not direct questions about the exam unless you're having technical difficulties downloading or something like that.

1. Download the file `FinalExamData1` from the class website. If you load it (be sure the command window is in the right directory, then type `load FinalExamData1`), two matrices, A and B will load- Matrix A is 8×4 and matrix B is 8×6.

   (a) Verify that the Col(A) and Col(B) have the same dimension.

   (b) Determine whether or not Col(A) and Col(B) are the same subspace of $\mathbb{R}^8$. Explain what you calculated and why it worked.
   (Hint: There are many ways you might do this.)

2. Let
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \\ 13 & 14 & 15 \\ 16 & 17 & 18 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

   (a) Compute the SVD of A and find the rank of A.

   (b) Compute $\hat{\mathbf{b}}$ and $\mathbf{z}$ from the the orthogonal decomposition $\mathbf{b} = \hat{\mathbf{b}} + \mathbf{z}$, where $\hat{\mathbf{b}}$ is in the columnspace of A.

   (c) Verify (in Matlab) that $\mathbf{z} \in \text{Null}(A^T)$.

   (d) Compute the distance from $\mathbf{b}$ to Col(A).

   (e) Use the pseudoinverse (from the SVD) to compute the least-squares solution to $A\mathbf{x} = \mathbf{b}$.

3. For the third problem, download the other dataset from our class website, `ExamFaceData.mat` (Right-click, and "Save Link As...").

When we load the data into Matlab, you'll see a large array `Y1` that will be 62700×32, corresponding to 32 column vectors in $\mathbb{R}^{62700}$. Each column vector corresponds to the photo of a face that is 285×220 (or 62700 pixel elements altogether).

Use the script file from class, `SampleAuthor.m` (which has been modified slightly- Use the link from the Final Exam stuff, not the earlier version) to do the following:

(a) Load the data

(b) Compute the average face and plot it.

(c) Find the SVD of the mean-subtracted data.

(d) Visualize the first four columns of U (in the SVD) as "photos" and plot them.

(e) Project the 32 faces to two dimensions and plot the result.

(f) (Just for fun) Watch a movie of the faces!

HINT: The amount you actually have to do here is very minimal. The script will basically do everything for you if you get the first few lines correct.