Homework Solutions, Matlab Assignment 1

The homework was to solve the following exercises using Matlab. Students were allowed to summarize the answers by hand.

- Section 1.2, #34
- Section 1.4, #40, 42
- Section 1.7, #41, 43

Here are the solutions:

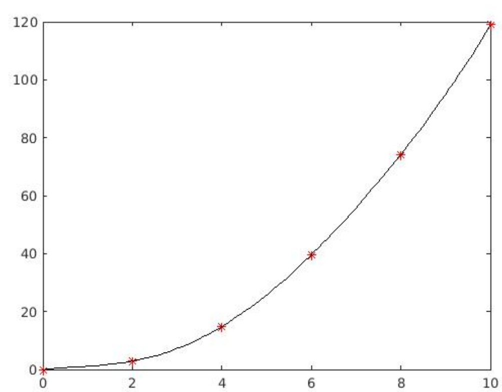
1.2.34 We want to find an interpolating polynomial between velocity (V) and force (F), then evaluate that polynomial when the velocity is 7.5. Here is the solution in Matlab:

```
F=[0 2.9 14.8 39.6 74.3 119]';
A=[ones(6,1) V V.^2 V.^3 V.^4 V.^5]
C=rref([A F]);       %The last column is the vector of coefficients for the poly
coefs=C(6:-1:1,end) %These are the coefficients for the polynomial
% These commands will create points to plot (optional):
cc=linspace(0,10);
yy=polyval(coefs,cc);
% Plot of the data and our curve
plot(V,F,'r*',cc,yy,'k-');
% Evaluate the polynomial at a velocity of 7.5:
polyval(coefs,7.5)
```

The coefficients are (in reverse order and rounded):

$$[0.0026 \qquad -0.07 \qquad 0.66 \qquad -1.19 \qquad 1.71 \qquad 0]$$

The value at 7.5 is approximately 64.8. Here is a plot of the data and our curve:



The idea in 40 and 42 is the connection between the span and the pivots. Here, we will enter the matrices and check the row reduced echelon form of each.

1.4.40
```
>> A=[8 11 -6 -7 13;-7 -8 5 6 -9;11 7 -7 -9 -6;-3 4 1 8 7];
>> rref(A)
```

We see pivots in columns 1, 2, 4 and 5. Column three is not a pivot column. However, there is a pivot in every row, so the columns of A do span $\mathbb{R}^4$.

1.4.42 From exercise 40, we saw that column three is not a pivot column, so it could be deleted from the matrix A without any effect on the spanning set. We cannot delete more than one column, because then we would lose a pivot row.

1.7.41 Use as many columns of A as possible to construct a matrix B with the property that the equation $B\mathbf{x} = \vec{0}$ has only the trivial solution. Verify by solving $B\mathbf{x} = \vec{0}$.

SOLUTION: By row reducing the matrix A, we see that columns 3 and 4 are not pivot columns, while columns 1, 2, and 5 are pivot columns. Therefore, to construct B we should take columns 1, 2, and 5 from the matrix A, then $B\mathbf{x} = \vec{0}$ has only the trivial solution.

```
>> A=[8 -3 0 -7 2;-9 4 5 11 -7;6 -2 2 -4 4;5 -1 7 0 10]
>> rref(A)
```

1.7.43 With A and B as before, select a column $\mathbf{v}$ of A that was not used in the construction of B, and determine if $\mathbf{v}$ is in the set spanned by the columns of B.

SOLUTION: The RREF of A in the first part was shown to be:

$$\text{RREF}(A) = \begin{bmatrix} 1 & 0 & 3 & 1 & 0 \\ 0 & 1 & 8 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So, if $\mathbf{v} = \mathbf{a}_3$, then we see that

$$\mathbf{a}_3 = 3\mathbf{a}_1 + 8\mathbf{a}_2 = 3\mathbf{b}_1 + 8\mathbf{b}_2$$

Similarly, if $\mathbf{v} = \mathbf{a}_4$, then we see that

$$\mathbf{a}_4 = \mathbf{a}_1 + 5\mathbf{a}_2 = \mathbf{b}_1 + 5\mathbf{b}_2$$

Therefore, both columns are in the span of the column of B.