Matlab for Math 240

1 Introduction and Set Up

Matlab (short for Matrix Laboratory) was initially designed as a front end for linear algebra, and has evolved into one of the main pieces of software for applied mathematicians and engineers.

Matlab is installed on all of the computers in the Mathematics Computer Lab, so you'll need an account on them to work with Matlab (see your instructor if you need an account or have forgotten your password).

To **find the Matlab icon**, go to the icon in the upper left part of your screen- This should allow you to search your computer. Type in Matlab and you should see the icon (if you don't, ask the student lab assistant or send your instructor a quick email).

1.1 Matlab for your home machine

Unfortunately, our site license doesn't allow for students to use the College license on personal computers. If you're planning on either applied mathematics or engineering in your future, you might look into a student edition of Matlab (at the time of this writing, Matlab alone is about $49, and with a wide array of numerical toolboxes that will take you into graduate school, the cost is $99).

Alternatively, there is a software package called "Octave" that was designed to be a free version of Matlab in the sense that most Matlab commands will work on it. In the past, the documentation was hard to follow, so we haven't used this for the class yet. It is currently being updated, so if you are an experienced computer user, you might try it out.

1.2 Opening and Saving Matlab

When you open Matlab for the first time, you will see several windows. The center window is where you type commands (so it is the command window). In that center window, you should see a bar with the line:

New to Matlab? See resources for <u>Getting Started</u>

If you click on the "Getting Started" link, a browser window will come up. If you look at the bottom of the link, you'll find two videos that you should watch.

Note: These videos are available on YouTube, but the "Getting Started" for the new version has been compressed to the point where you cannot read the commands being typed. You can view the older version on YouTube- The link is below (and on the class website)

`https://www.youtube.com/watch?v=TDWB6KZ1oWo`

The second video tutorial is also worth watching, although the "development interface" (or Matlab desktop) has changed quite a bit in the new version. Many features are the same, though (the link is on the class webpage).

Section 1.1 in Matlab

In Section 1.1, we defined a matrix and a vector, and we decided how to access elements of those. For example, look at Example 1 on p. 5. We used row operations on the matrix below to solve the corresponding system.

$$\left[\begin{array}{rrr|r} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{array}\right]$$

In Matlab, we can enter the matrix as the following (I will store the matrix in variable `A` in Matlab. The matrix is entered row-wise. You denote the end of a row with a semicolon or you can "enter". Here are the two examples that do the same thing. Type the following into the command window of Matlab.

```
%This is a comment- Matlab does not evaluate it.
%Enter the matrix A (two ways below, just choose one, but either works)
A=[1 -2 1 0;0 2 -8 8;-4 5 9 -9];
% The semicolon at the end of the line "suppresses output".
% You might leave it out to see what happens.

A=[1 -2 1 0
0 2 -8 8
-4 5 9 -9];

% To row reduce the matrix stored in A, type:
B=rref(A)
```

You should see that the matrix B is:

$$B = \left[\begin{array}{rrr|r} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

And similarly, if we check Example 3, we want to see if the system they give is consistent. That is, we want to see what the row reduced form of the augmented matrix is. Type the following into the command window:

```
A=[0 1 -4 8;2 -3 2 1;5 -8 7 1];
rref(A)
```

Matlab gives us the following matrix:

$$\left[\begin{array}{rrr|r} 1 & 0 & -5 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

Is this matrix row equivalent to the matrix given in the textbook? Explain.

Section 1.2 in Matlab

Here's exercise 33. We want to find an interpolating polynomial $y = a_0 + a_1 t + a_1 t^2$ for the data: $(1, 12), (2, 15), (3, 16)$. Therefore, we want to find a_0, a_1, a_2 so that the following system of equations is solved:

$$\begin{aligned} a_0 + a_1(1) + a_2(1)^2 &= 12 \\ a_0 + a_1(2) + a_2(2)^2 &= 15 \\ a_0 + 1_1(3) + a_2(3)^2 &= 16 \end{aligned}$$

In Matlab, we'll write and row reduce the augmented matrix. Type the following into the command window:

```
A=[1 1 1 12;1 2 4 15;1 3 9 16];
rref(A)
```

The row reduced form of the matrix is then given as

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

Therefore, the polynomial is $y = 7 + 6t - t^2$.

Section 1.4 in Matlab

The Matlab exercises here are 37-42. Here, we'll do Exercise 39 and Exercise 41.

39. Determine if the columns of the matrix span $\mathbb{R}^4$.

```
A=[12 -7 11 -9 5
-9 4 -8 7 -3
-6 11 -7 3 -9
4 -6 10 -5 12];

rref(A)
```

The result was that columns 1, 2, 3, and 5 were the pivot columns, and column 4 corresponded to a free variable. But we do have a pivot in every row, so the answer to the question is "Yes, these columns do span $\mathbb{R}^4$".

41. Find a column of the matrix in Exercise 39 that can be deleted and yet have the remaining matrix columns still span $\mathbb{R}^4$.

SOLUTION: From the RREF of A that we found above, we see that column 4 (the one corresponding to the free variable) can be removed, and the remaining columns will still span $\mathbb{R}^4$.

Section 1.7 in Matlab

42. We want to use as many columns of A as possible to construct a matrix B with the property that $B\mathbf{x} = \vec{0}$ has only the trivial solution. Solve $B\mathbf{x} = \mathbf{0}$ to verify your work:

```
>> A=[12 10 -6 -3 7 10
-7 -6 4 7 -9 5
9 9 -9 -5 5 -1
-4 -3 1 6 -8 9
8 7 -5 -9 11 -8];

>> rref(A)

ans =

     1     0     2     0     2     0
     0     1    -3     0    -2     0
     0     0     0     1    -1     0
     0     0     0     0     0     1
     0     0     0     0     0     0
```

We construct B by using columns 1, 2, 4 and 5:

```
B=A(:,[1,2,4,5]);    % The colon in the row position means "all rows"
```

44. With A and B as in Exercise 42, select a column $\mathbf{v}$ of A that was not used in the construction of B and determine if $\mathbf{v}$ is in the set spanned by the columns of B.

SOLUTION: From our previous calculations, if we choose $\mathbf{v}$ to be column 3 from matrix A, then the RREF of A tells us that $\mathbf{v}$ is a linear combination of the first two columns. In fact:

$$\mathbf{v} = 2\mathbf{a}_1 - 3\mathbf{a}_2$$

which we can check in Matlab:

```
>> 2*A(:,1)-3*A(:,2)

ans =

    -6
     4
    -9
     1
    -5
```