

## Content for Exam 2, Linear Algebra

Exam 2 will cover material from 2.2-2.3 (Inverse matrices), 3.1-3.3 (determinants), 4.1-4.6 (vector spaces) and 4.9 (Markov chains). Just like last time, no notes or calculators will be allowed, and you have 50 minutes.

### Sections 2.2-2.3

#### 1. Skills

- Compute the inverse of a  $2 \times 2$  matrix  $A$  directly (Theorem 4)
- Compute the inverse of an  $n \times n$  matrix using row reduction.
- Compute the elementary matrix for a given row operation.
- Solve a matrix equation using inverses.

#### 2. Know the Invertible Matrix Theorem (Theorem 8)

That is, you do not need to be able to list all of the parts, but given a prompt, be able to finish the statement so that it is equivalent to  $A$  being invertible. For example, “What is true about the columns of  $A$ ?” Answer might be: Columns are linearly independent, Columns are pivot columns, Columns span  $\mathbb{R}^n$  (any of these).

#### 3. Theorems

- Understand Theorem 5, 6, 7 (be able to compute using them).
- You do not need to know Theorem 9 (p 131).

### Sections 3.1-3.3

#### 1. Skills

- Be able to compute determinants using a cofactor expansion along any row or column.
- Compute a determinant for upper or lower triangular matrix.
- Be able to compute a determinant by first performing row reduction.
- Use Cramer’s Rule to solve a system,
- Compute the volume of a parallelepiped, area of a parallelogram.

#### 2. Properties of the determinant. For the following, assume $E, A, B$ are square matrices. For the last item, assume $A$ is invertible.

##### (a) Elementary matrices:

- $E$  corresponding to a row swap:  $\det(E) = -1$
- $E$  corresponding to multiplying a row by  $k$ :  $\det(E) = k$
- $E$  corresponding to  $kr_j + r_i \rightarrow r_i$ :  $\det(E) = 1$

##### (b) General properties:

- $A$  is invertible only if  $\det(A) \neq 0$ .
- If  $A$  is  $n \times n$ , then  $\det(kA) = k^n \det(A)$ .
- $\det(AB) = \det(A)\det(B)$
- $\det(A^T) = \det(A)$
- $\det(A^{-1}) = 1/\det(A)$

#### 3. Theorems

- Theorems 1, 2, 3 and 9 are used in computation. You do not need to memorize these, you can just use them.
- You should know Theorems 4, 5, 6, 7. These are summarized in the properties, and Theorem 7 is Cramer's Rule.
- Theorem 8 (A formula for  $A^{-1}$  involving the "adjoint" will not be on the exam, neither will the adjoint. Cofactors (book notation:  $C_{ij}$ ) will be used in computing determinants.
- Theorem 10 will not be on the exam (the area of an image).

## Vector Spaces, (4.1-4.6, 4.9)

1. You don't need to memorize the 10 axioms on page 217.
2. Be familiar with some template vector spaces:

$$\mathbb{R}^n, \quad \mathbb{P}_n, \quad \mathbb{P}, \quad C[a, b], \quad M_{m \times n}$$

3. Know these definitions: A subspace, a linearly independent set, a basis, the coordinates of  $\mathbf{x}$  (with respect to a given basis), the dimension of a subspace, an isomorphism, the rank of a matrix. The four fundamental subspaces associated with a matrix  $A$  (be able to define each one), the kernel of a transformation, the change of coordinates matrix. A steady state vector for a Markov Chain. A regular stochastic matrix.
4. Theorems for computation: 1, 4, 5, 6, 7, 9, 10, 11, 13, 18 (from 4.9).  
These are theorems that you should know for computational purposes (you might think of them as "basic facts").
5. Theorems to know: 2 (Null space is a subspace), 3 (Col space is a subspace), 8 (Isomorphism for Isomorphic Spaces), 12 (The basis theorem), 14 (The rank theorem).

6. Skills:

- Prove that a given set is or is not a subspace.
- Given a matrix  $A$ , be able to compute a basis for the column space, the null space and the row space (not the null space of  $A^T$ ).
- Find the kernel of a given transformation and describe the range of the transformation.
- Understand how row operations effect the the 4 fundamental subspaces (for example, the subspaces for a matrix  $A$  versus its RREF,  $B$ ).
  - Row operations do not effect the relationship among the columns of  $A$ , but they do effect the column spaces (the column spaces of  $A$ ,  $B$  may not be the same).
  - Row operations do effect the relationship among the rows of  $A$ , but the row spaces of  $A$ ,  $B$  are the same.
  - Row operations do not effect the set of solutions to  $A\mathbf{x} = \mathbf{0}$ , so the null spaces of  $A$ ,  $B$  are the same.
- Find the coordinates of a vector given a basis (both in  $\mathbb{R}^n$  using the change of coordinates matrix, and for vector spaces that are not  $\mathbb{R}^n$ , like  $\mathbb{P}_n$ ).
- Be able to compute the dimension of a vector space.
- Be able to compute the rank of a matrix. Use that to compute the dimensions of the four fundamental subspaces.
- Understand what it means to say that two vectors spaces are **isomorphic**.
- Theorem: Any finite dimensional vector space  $V$  (with dimension  $n$ ) is isomorphic to  $\mathbb{R}^n$  using the coordinate mapping as the isomorphism.