Review Material, Exam 3

The exam will cover the material from Sections 4.9, 5.1-5.5 (except 5.4), and 6.1-6.4 and some of 6.5 (on a handout in class). In these sections, we have studied eigenvalues and eigenspaces, diagonalization, and geometry in $\mathbb{R}^n$ (angle, distance, projection).

Key Definitions

Probability vector, stochastic matrix, Markov chain, steady state vector (or equilibrium vector)

Eigenvalue, eigenvector, eigenspace, characteristic polynomial (and equation), algebraic and geometric multiplicity of an eigenvalue, defective matrix. A is similar to B, diagonalizable.

Complex conjugate, polar form of a complex number, magnitude (or length) for a complex number, argument for a complex number. Eulers Formula (with the complex exponential). Form of "diagonalization" when the eigenvalues are complex.

Inner product (dot product), norm, distance, angle between vectors, orthogonal vectors, an orthogonal set, an orthogonal matrix (careful with this one), orthonormal set, orthonormal basis, orthogonal projection of one vector onto another, orthogonal projection of one vector onto a subspace.

Key Theorems

You should be able to prove these. I would state them for you, so you do not need to memorize the statement of the theorem.

Chapter 5:

Theorem 1 (Eigenvalues of a triangular matrix),
Theorem 2 (When are evecs linearly independent)
Theorem 4 (Similar matrices have the same eigenvalues)
Relationship of the eigenvalues (or diagonalizability) to the invertibility of a matrix.

Chapter 6:

Theorem 2 (Pythagorean Theorem)
Theorem 3 (Null(A) is orthogonal to Row(A))
Theorem 4 (Orthogonal non-zero vectors are linearly independent)
Theorem 5 (Given an orthogonal basis and a vector, compute the coordinates)
Theorem 6 (How to check if U has orthonormal columns)
Theorem 7 (U is a rigid rotation)
Theorem 10 (Matrix form for a projection)

I wont ask you to prove Theorem 8 (Orthogonal Decomposition Theorem) or Theorem 9 (The Best Approximation Theorem), but you should understand how they are used (they are key theorems for later). Ill ask you to use Theorem 11 (Gram-Schmidt), but you will not be asked to prove it.

Chapter 5 skills

- Recall how to work with the determinant (used for eigenvalues, summarized again in Section 5.2).
- Be able to compute basic operations using complex numbers (in particular, multiplica- tion and division). Be able to convert between complex numbers and the polar form using Eulers Formula (not in text). Use the handout from class as your guide.

- Know the three equations associated with eigenvalues and eigenvectors:

$$A\mathbf{x} = \lambda\mathbf{x} \qquad (A - \lambda I)\mathbf{x} = \mathbf{0} \qquad \det(A - \lambda I) = 0$$

  and know which one to use (either to compute something or to prove something).

- Find the characteristic equation and eigenvalues of a 2×2 matrix. Find the eigenvalues of a triangular matrix. Find a basis for an eigenspace. (NOTE: For matrices larger than 2×2, the eigenvalues would either be given, the matrix would have a special form, or just compute the characteristic polynomial).

- Be able to factor a matrix (PCP^{-1}) if it has complex eigenvalues (Theorem 9, p. 340)

- If $C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, explain the "action" of C using scaling r and CCW rotation by angle θ.

- Given A (bigger than 2×2), be able to factor it using a combination of techniques from the previous two items ($A = PDP^{-1}$ and $A = PCP^{-1}$).

Chapter 6 skills

- Computations related to the geometry of $\mathbb{R}^n$: Compute length of vector, distance be- tween vectors, angle between vectors. Check a set for orthogonality. Normalize a vector.

- Compute the orthogonal projection of a vector onto a vector, project a vector onto a subspace (in particular, a line or plane that includes the origin).

- Decompose a vector into a component in the direction of u and a component orthogonal to u, or as the sum of a vector in subspace W and a vector in $W^{\perp}$.

- Be able to use Gram-Schmidt to find a comparable set of vectors that are orthogonal.

- Understand the relationship of the four fundamental subspaces for a matrix A to the matrix equation $A\mathbf{x} = \mathbf{b}$, especially when there is no solution.