M244
Sample 02

Final Exam

Name:__________

Show all your work! You may not use your text, colleagues or a calculator. A table of Laplace transforms and the Poincare Diagram is provided.

1. Solve $\mathbf{x}' = A\mathbf{x}$ for the given matrix A:
   (a) $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$
   (b) $A = \begin{bmatrix} 1 & -1 \\ 5 & -3 \end{bmatrix}$

2. Convert the following system to an equivalent 2d order DE:
$$\mathbf{x}' = \begin{bmatrix} -1 & -4 \\ 1 & -1 \end{bmatrix} \mathbf{x}$$

3. Convert the following 2d order DE to an equivalent system of first order equations (do not solve): $y'' + 3y' + 5y = 0$

4. Find the radius of convergence for the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n(n+2)}$

5. Find the recurrence relation between the coefficients of the power series solution to the following at the given value of x_0:
$$xy'' + y' + xy = 0, \qquad x_0 = 1$$

6. Solve the following for $Y(s)$ (the Laplace transform of $y(t)$), but do NOT invert!
$$y'' + 2y' + 2y = \cos(3t) + \delta(t-2) \qquad y(0) = 1, y'(0) = 2$$

7. Invert the Laplace transform: $\dfrac{e^{-2s} - e^{-3s}}{2s^2 + s + 2}$

8. Guess the final form of the particular solution for each DE using the Method of Undetermined Coefficients (do NOT solve for the coefficients)
   (a) $y'' + 3y' = 2t^4 + t^2 e^{-3t} + \sin(3t)$
   (b) $y'' + y = t(1 + \sin(t))$

9. If y_1, y_2 are a fundamental set of solutions to $t^2 y'' - 2y' + (3 + t)y = 0$, and if $W(y_1, y_2)(2) = 3$, then find the value of $W(y_1, y_2)(4)$.

10. A spring with a mass of 3 kg is held stretched 0.6 m beyond its natural length by a force of 20 N. If the spring begins at its equilibrium position but a push gives it an initial velocity of 1.2 m/s, find the IVP modeling the position of the mass at time t (assume no damping). Do NOT solve.

11. A tank initially contains 10 kg of salt in a tank that contains 100 gallons of brine. Brine is flowing into the tank at a rate of 5 gallons per minute, and contains 2 kg of salt per gallon. The well mixed brine is pumped out of the tank at a rate of 6 gallons per minute. Write the IVP that gives the amount of salt in the tank at time t (do NOT solve).

12. Solve:

    (a) $(9x^2 + y - 1)\,dx - (4y - x)dy = 0$ with $y(1) = 3$.

    (b) $y' = 2x/(y + x^2 y)$

    (c) $ty' - y = t^2 e^{-t}$, $t > 0$.

13. Consider the system: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \alpha & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

    Using the Poincaré Diagram (you might sketch it), describe how changing α changes the classification of the origin.