1. Use e^{iat} to help compute the Laplace transform of $\cos(at)$.

   SOLUTION:

   Since $\mathrm{e}^{iat} = \cos(at) + i\sin(at)$, then

$$\mathcal{L}(\mathrm{e}^{iat}) = \mathcal{L}(\cos(at)) + i\mathcal{L}(\sin(at))$$

   so we compute the Laplace transform of e^{iat}, then take the real part for our answer:

$$\mathcal{L}(\mathrm{e}^{iat}) = \int_0^\infty \mathrm{e}^{-st}\mathrm{e}^{iat}\,dt = \int_0^\infty \mathrm{e}^{-(s-ai)t}\,dt = -\frac{1}{s-ai}\mathrm{e}^{-(s-ai)t}\Big|_0^\infty =$$

$$0 + \frac{1}{s-ai} = \frac{s+ai}{s^2+a^2}$$

   And the real part is $s^2/(s^2+a^2)$.

2. Same idea, but we'll compute $\mathcal{L}(\mathrm{e}^{at}(\cos(bt) + i\sin(bt))$.

$$\int_0^\infty \mathrm{e}^{-st}\mathrm{e}^{(a+ib)t}\,dt = \int_0^\infty \mathrm{e}^{-((s-a)+bi)t}\,dt = -\frac{1}{(s-a)-bi}\mathrm{e}^{-((s-a)-bi)t}\Big|_0^\infty =$$

$$\frac{1}{(s-a)-bi} = \frac{(s-a)^2+b^2}{(s-a)^2+b^2}$$

   We take the real part.

3. Exponential order practice:

   (a) $\sin(t)$: We can use e^t, for $t > 0$ (see graph)

   (b) $\tan(t)$ has a vertical asymptote at $t = \pi/2$ (and multiples of π thereafter), so it is NOT of exponential order.

   (c) $t^3 = \mathrm{e}^{\ln(t^3)} = \mathrm{e}^{3\ln(t)} \leq \mathrm{e}^{3t}$, so this is of exponential order.

   (d) e^{t^2} is not of exponential order, since the exponent is of order 2.

   (e) $5^t = \mathrm{e}^{\ln(5^t)} = \mathrm{e}^{t\ln(5)}$

   (f) t^t is not of exponential order:

$$t^t = \mathrm{e}^{\ln(t^t)} = \mathrm{e}^{t\ln(t)}$$